Roll No. 22311232000 Exam Code : M-25

Subject Code—64499

B. Sc. EXAMINATION

(Batch 2022 Onwards)

(Sixth Semester)

MATHEMATICS

CML-607(i)

Real & Complex Analysis

Time : 3 Hours *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Give an example of a pseudo metric space which is not a metric space.

(b) Let A and B are two subsets of a maetric space X then :

(i) $A^o \cup B^o \subset (A \cup B)^o$

(ii) $(A \cap B)^o = A^o \cap B^o$

P.T.O.

(c) Prove that continuous image of a compact metric space is compact.

(d) Find k if the function $f(z) = r^2 \cos 2\theta + i r^2 \sin k\theta$ is analytic.

(e) Evaluate $\lim\limits_{x \to i} \dfrac{z^{10}+1}{z^6+1}$.

(f) Check the derivability of the function $f(z) = |z|$ at origin in complex plane.

(g) Discuss the convergence of the improper integral $\int_0^\infty \frac{1}{x^n} dx, (a > 0)$.

(h) State Frullani's Theorem.

Section I

2. (a) Prove that $d(x, y) = \dfrac{|x-y|}{1+|x-y|}$ is a metric on R.

(b) If 'a' is a limit point of a subset A of a metric space (X; d), then there is a sequence $<a_n>$ of points of A all distinct from 'a' which converge to 'a'.

3. (a) Prove that the metric space (R; d); is complete, where d denotes the usual metric.

(b) State and prove Cantor's Intersection Theorem.

Section II

4. (a) If (X, d) and (Y, d^*) be metric spaces and f be a function of X into Y. Then f is continuous iff $f^{-1}(G)$ is open in X whenever G is open in Y.

(b) State and prove Baire's Category Theorem.

5. (a) A metric space is sequentially compact iff it has the Bolzano-Weierstrass property.

(b) A metric space (X, d) is compact iff every collection of closed subsets of X with finite intersection property has a non-empty intersection.

Section III

6. (a) Discuss the convergence of the integral

$$\int_0^\infty x^{n-1} e^{-x} dx.$$

(b) Examine the convergence of the integral

$$\int_0^\infty \frac{\sin x}{x^p} dx.$$

7. (a) Evaluate the integral $\int_0^\infty \frac{\tan^{-1} \alpha x}{x(1+x^2)} dx$,

where $\alpha \geq 0$.

(b) Show that :

$$\int_0^\infty \frac{\sin ax \sin bx}{x} dx = \frac{1}{2} \log \frac{a+b}{a-b}$$

where $a > b > 0$.

Section IV

8. (a) Show that the function $f(z) = |z|^2$ nowhere differentiable except at the origin.

(b) Show that the function $f(z) = |xy|$, $z = x + iy$ is not analytic at the origin, althrough Cauchy-Rieman equation are satisfied at that point.

9. (a) If $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2y}$ and $f(z) = u + iv$ is an analytical function of z, then find $f(z)$ in terms of z.

(b) The continuous one-valued function $f(z)$ is analysis in a domain D if the four partial derivatives u_x, u_y, v_x and v_y exist, are continuous and satisfy the C-R equations at a point of D.

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