Unit III

6. (a) Prove that a top space is connected iff it has no non-empty proper subset which is both open and closed. **7**

(b) Define first and second countable spaces. Show that every second countable space is first countable but the converse is not true. **7**

7. (a) Prove that separability and Lindelofness are independent concepts. **7**

(b) Show that a top space (X, J) is T_0 iff the closures of distinct points are distinct. **7**

Unit IV

8. (a) Prove that a top space (X, J) is regular iff for every point $x \in X$ and open set G containing x, $\exists$ an open set G* s.t. $x \in G^*$ and $\overline{G^*} \subseteq G$. **7**

(b) Show that a compact Hausdorff space is a T_4-space. **7**

Roll No. .... **Exam Code : J-21**

Subject Code—7761

M.Sc. EXAMINATION

(Third Semester)

MATHEMATICS

MAL-631

Topology

Time : 3 *Hours* *Maximum Marks* : 70

Note : Attempt *Five* questions in all. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Define closure of a set and show that : **2**

$$\overline{A \cap B} \neq \overline{A} \cap \overline{B}$$

(b) Define neighbourhood system of a point and prove that the intersection of two neighbourhoods of $x \in X$ is also a neighbourhood of x. **2**

(c) Show that usual topological space is not compact. **2**

(d) Define locally compact space. Also prove that a locally compact space need not be compact. **2**

(e) Show that every component of a locally connected space is an open set. **2**

(f) Prove that a finite space is T_1 iff it is discrete. **2**

(g) Show that normality is not a hereditary property. **2**

Unit I

2. (a) Show that a subset F of a top space (X, J) is closed iff its complement is open. **7**

(b) Discuss dense set in : **7**

(i) Indiscrete topological space

(ii) Cofinite topological space

(iii) Countable topological space.

3. (a) Prove that a family β of sets is a base for a topology for the set $X = U\{B : B \in \beta\}$ iff for every $B_1, B_2 \in \beta$ and every $x \in B_1 \cap B_2$, $\exists$ a set $B \in \beta$ s.t. $x \in B \subseteq B_1 \cap B_2$. **7**

(b) Show that a mapping f from X to X* is continuous iff $f(C(E)) \subseteq C^* f(E)\ \forall\ E \subseteq X$. **7**

Unit II

4. (a) Prove that a top space is compact iff any family of closed sets having FIP has a non-empty intersection. **7**

(b) Define countably compact topological space. Also show that a countably compact topological space has the Bolzano Weierstrass property. **7**

5. (a) Define continuous function. Also prove that the continuous image of a compact set is compact. **7**

(b) Show that a sequentially compact metric space is compact. **7**

9. (a) Prove that a normal space is completely regular iff it is regular. **7**

(b) Show that every projection $\Pi_\lambda : X \to X_\lambda$ on a product space $X = \Pi_\lambda X_\lambda$ is open and continuous. **7**

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