Roll No. 2200606/0047.

Exam Code : M-23

Subject Code—12001

M. Sc. EXAMINATION

(Batch 2016 Onwards)

(Second Semester)

MATHEMATICS

MAL-521

Abstract Algebra

Time : 3 Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

1. (a) Show that every nilpotent transformation is singular. 2

(b) Define the matrix M_n. Find index of nilpotence of M_4. 2

(c) Define companion matrix of $1 - 2x + x^7$. 2

(d) Define Simple module. Is every simple module cyclic ? 2

(e) Define an Artinian module and a ring. Give an example of each. 2

(f) Obtain the Smith normal form of $\begin{bmatrix} 0 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$ over ring of integers. 2

(g) Show the abelian group generated by x and y subject to $3x = 0$ and $5y = 0$. 2

Unit I

2. (a) What do you understand by Triangular form of a Transformation ? If all the characteristic roots of a transformation T acting on a vector space V are in F, then there exists a basis of V in which matrix of T is triangular. 7

(b) Let $T \in A(V)$ be a nilpotent transformation. Use T to write V as a direct sum of two of its invariant subspaces. 7

3. Show that two transformations are similar iff they have same invariants. Hence deduce that $\begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ are similar. 14

Unit II

4. (a) Let $h(x) = p_1(x)^{\alpha_1} p_2(x)^{\alpha_2} \ldots\ldots p_k(x)^{\alpha_k}$, where each $p_i(x)$ is irreducible, be the minimal polynomial of $T \in A(V)$. Use $h(x)$ to write V as a direct sum of its invariant subspaces. 7

(b) Let $p(x)$, be the minimal polynomial of $T \in A(V)$. If V as module is cyclic, then there exists a basis of V in which matrix of T is the companion matrix of $p(x)$. 7

5. Let V and W be two spaces over F and suppose that ϕ is an isomorphism from V to W. If $(xT)\phi = (x\phi)S$ for all $x \in V$, where $T \in A(V)$ and $S \in A(W)$, then T and S have same

elementary divisors. Hence deduce that two transformations are similar iff they have same elementary divisors. **14**

Unit III

6. Show that an R module M is Noetherian iff its every collection of submodules of M has a maximal element. Hence deduce that Z as Z module is Noetherian. **14**

7. (a) Show that $Hom_R(M, M)$, where M is a simple module, is a division ring. **7**

(b) Show that R is a Noetherian ring iff R[x] is Noetherian ring. Does the result hold for artinian rings also ? **7**

Unit IV

8. State and prove Wedderburn-Artin theorem. **14**

9. (a) Write a procedure to obtain the Smith normal form of a matrix over a principal ideal domain. **7**

(b) Find the abelian group generated by x, y, z subject to $5x+9y+5z=0$, $2x+4y+2z=0$, $x+y-3z=0$. 7