Roll No. .... **Exam Code : M-26**

Subject Code—30604

M.Sc. (UTD and Affiliated Colleges) (Under NEP 2020) EXAMINATION

(Second Semester)

(Batch 2025 Onwards)

MATHEMATICS

U25MAT204T

Complex Analysis-II

Time : 3 Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) State Ascoli Arzela theorem.

(b) Construct the canonical product associated with sequence of negative integers.

(c) Find the order of the function $e^{z^{\lambda}}, \lambda,$ being +ve integer.

(d) Define Bloch's Constant and Landau's Constant.

(e) State Schottky's theorem.

(f) Define Green function.

(g) Show that :

$\Gamma(z+1) = z\Gamma(z), z \neq 0, -1, -2, ...$ 7×2=14

Unit I

2. (a) Show that the series :

$$\frac{1}{2} + \frac{z}{4} + \frac{z^2}{8} + \frac{z^3}{16} - ......$$

and $\frac{1}{2-i} + \frac{z-i}{(2-i)^2} + \frac{(z-i)^2}{(2-i)^3} + ...$

are analytic continuation of each other. 7

(b) State and prove Hurwitz's theorem. 7

3. State and prove Montel's theorem. 14

Unit II

4. State and prove Hadamard Factorization theorem. **14**

5. (a) Define infinite product. Also, prove that

$$\sin \pi z = \pi z \prod_{n=1}^{\infty}\left(1-\frac{z^2}{n^2}\right). \qquad 7$$

(b) State and prove Jenson's formula. 7

Unit III

6. (a) State and prove Great Picard theorem. 7

(b) Let f be analytic in $D=\{z:|z|<1\}$ such that $f(0) = 0, f'(0) = 1$ and $f(z) \leq M$ for all z in D. Then

$$M \geq 1 \text{ and } f(D) \supset B\left(0;\frac{1}{6M}\right). \qquad 7$$

7. State and prove Bloch's theorem. **14**

Unit IV

8. (a) Define Gamma function and derive Gauss's formula from it. Also show that : 7

$$\sin \pi z = \frac{\pi}{\Gamma(z)\Gamma(z-1)};$$

$z \neq 0, \pm 1, \pm 2, \ldots\ldots$

(b) Let G and Ω be regions such that there is a one-one analytic function f of G onto Ω :

let $a \in G$ and $\alpha = f(a)$. If g_a and γ_α are the Green's functions for G and Ω with singularities 'a' and 'α' respectively, then 7

$g_a(z) = \gamma_\alpha(f(z))$.

9. State and prove Riemann mapping theorem. 14

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