Roll No. .... Exam Code : M-24

Subject Code—18006

M. Sc. Mathematics (Second Semester) (Batch 2020 Onwards)/(Dual Degree)

B. Sc. (Hons.) Mathematics (Eighth Semester) (Batch 2017 Onwards) EXAMINATION

ADVANCE NUMERICAL METHODS

MAL-526/MML-726

Time : 3 *Hours* *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

Compulsory Question

1. (a) Write down Bessel and Everett's interpolation formulas.

(b) Use Gauss backward interpolation formula to compute the value of $f(1.2)$

x :	0	1	2	3
$f(x)$:	1	2	1	10

(c) Find $\frac{dy}{dx}$ at $x = 0.1$ from the following table :

x :	0.1	0.2	0.3	0.4
y :	0.9975	0.9900	0.9776	0.9604

(d) Evaluate the integral $\int_0^1 x^2 e^{-x^2} dx$ by using Gauss-Legendre 2-points formula.

(e) State the necessary and sufficient conditions for convergence of Gauss-Seidel and SOR method.

(f) Using Rayleigh power method to find the largest eigenvalue and corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 10 & 6 & 7 \\ 1 & 7 & -2 \\ 2 & 2 & 2 \end{bmatrix}$$ with initial approximation $[1, 1, 1]'$. Perform only two iterations.

(g) State stability condition for Runge Kutta method.

Unit I

2. Apply Hermite interpolation formula to obtain a cubic polynomial which satisfies the following specifications :

x :	0.1	0.2
$f(x)$:	0.201	0.408
$f'(x)$:	2.03	2.12

3. Obtain the value of $y(2.4)$ for the following data with cubic splines :

x :	− 1	1	3	5
y :	2	4	11	23

Use end conditions $f''(-1) = 0$ and $f''(5) = 0$.

Unit II

4. Calculate the approximation to the double integral $\int_1^2\int_1^2 \frac{1}{(1+x+y)} dx\, dy$ with the aid of Simpson method by taking $h = 0.5$ and $k = 0.25$.

5. Use Euler-Maclaurin formula to compute the value of the integral $\int_1^2 e^{-x^2} dx$. Divide the interval into ten equal parts and use up to third derivative terms only.

Unit III

6. Derive the convergence condition for Gauss-Seidel method.

7. Use NR method with initial approximations $x_0 = y_0 = -0.5$, to compute a complex root of the following complex equation :

$$F(z) = z^3 - 4i\, z^2 - 3e^z = 0,$$

where the variable z is complex.

Unit IV

8. Use Adams Moulton predictor-corrector method to compute $y(0.4)$ from differential equation $\frac{dy}{dx} = y^2 - x^2$ and the following values :

x :	0	0.1	0.2	0.3
y :	1	1.11	1.25	1.42

9. Compute the approximate solution of the following differential equation using finite difference method,

$$xy'' - y' + 2xy = x; \qquad 1 \le x \le 2$$

subject to boundary conditions

$$y(1) = 1, \qquad y(2) = 3.$$

Subdivide the interval [1, 2] into four equal parts.