Roll No. .... Exam Code : M-23

Subject Code—12006

M. Sc. (Mathematics) (Second Semester) (Batch 2016 Onwards) (Advance Numerical Methods) (Common with Dual Degree B.Sc. (Hons.)) (Mathematics) (Eighth Semester)

EXAMINATION

MAL-526/MML-726

Time : 3 *Hours* *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** consists of seven short answer type questions is compulsory. All questions carry equal marks. Use of scientific calculator is allowed.

1. (a) Use Chebyshev polynomials to compute the best lower order approximation for the polynomial $3x^4 + 5x^3 - x + 1$ in the domian $-1 \leq x \leq 1$. Also, compute the error bound in this approximation. **2**

(b) The population of a certain town (as obtained from census data) is shown in the following table :

	Year	**Population**
x_{-1}	1981	12.92 → y_{-1}
x_0	1991	16.46
x_1	2001	21.14
x_2	2011	25.35 (in millions)

Find the estimate for the population in the year 1995 using Sterling interpolation method. 2

(c) Find $\frac{dy}{dx}$ at $x = 0.1$ from the following table : 2

x	y
0.1	0.9975
0.2	0.9900
0.3	0.9776
0.4	0.9604

(d) Evaluate the integral $\int_0^1 \frac{e^x}{1+\sin(x)} dx$ by using Gauss-Legendre 2-points formula. 2

(e) Using Rayleigh power method to find the largest eigen value and corresponding eigen vector of the matrix $\begin{bmatrix} 1 & 1 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 2 \end{bmatrix}$ with initial approximation $[1, 1, 1]'$. Perform only two iterations. 2

(f) Rewrite the following system such that it satisfies the convergence condition of Gauss-Seidel method :

$$\begin{bmatrix} 2 & -1 & 4 \\ 1 & 3 & -1 \\ -5 & 1 & -3 \end{bmatrix} X = \begin{bmatrix} 7 \\ -3 \\ -9 \end{bmatrix}.$$ 2

(g) Write down second-order Runge-Kutta method with minimum truncation error bound. 2

Unit I

2. Obtain the value of $y\,(1.3)$ for the following data by using cubic splines with natural end conditions : **14**

x	y
0	2
1	− 6
2	− 8
3	2

3. Compute the value of $f(-0.5, 4)$ for the following set of data points for the function $f(x, y)$:

$x \backslash y$	1	2	5
−1	2	5	26
0	1	4	25

14

Unit II

4. Apply Trapezoidal rule to compute the integral

$$\int_0^{0.6}\int_0^{0.5} e^{x+y}\,dx\,dy$$ by taking two equal subintervals for the variable x and three equal subintervals for the variable y. Also, find the exact solution. **14**

5. In an experimental setup, we have following input and output values. Compute the approximation for the output value if the input value is 1.35. Find the input value for maximum output. Also find that maximum output value. **14**

Input	**Output**
0.5	2.4
1.0	5.7
1.5	12.8
2.0	6.3

Unit III

6. Compute the solution of non-linear system :

$$\sin(xy) - x^2 - 5x - y = 0,$$

$$\cos(x+y) + y^2 - x - 6y + 2 = 0$$

P.T.O.

with the help of iterative method. Use an initial approximation (0, 0). **14**

7. Derive the convergence condition for Gauss-Seidel method. **14**

Unit IV

8. Use classical Runge-Kutta method of order 4 to find the values of y (0.1) and z (0.1).

$$\frac{dy}{dx} = z + x,$$

$$\frac{dz}{dx} = y^2 + x + z,$$

$$y(0) = 0, \ z(0) = 0.$$ **14**

9. Use finite difference approximation to find the values of y (0.25), y (0.5), y (0.75) for the following BVP :

$$y'' - 2xy' + y = x^2; \ y(0) = 0, \ y(1) = 0.$$ **14**