Roll No. .... Exam Code : N-21

Subject Code—11205

M.Sc. EXAMINATION

(Main & Re-appear) (For Batch 2019 Onwards)

(Second Semester)

MATHEMATICS

MAL-526

Advance Numerical Methods

Time : 3 *Hours* *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Define SOR and SUR method.

(b) What do you mean by the ill conditioned system ?

(c) Write on the following :

(i) Newton-Cotes integration

(ii) Adaptive Quadrature method.

(d) Write a note to compare efficacy of Stirling formula and Bessel formula.

(e) Define main features of the Hermite's interpolation formula.

(f) Compare the linear spline, quadratic splines and cubic splines on the basis of discontinuity in derivatives and which of the method is more suitable.

(g) In numerical differentiation discuss the dependency of truncation error and rounding errors on the interval size h.

Unit I

2. (a) Derive the Everett's formula from the Bessel's formula.

(b) From the following data find the value of exp(1.13) using Gauss's forward formula :

x	$\exp(x)$
1.00	2.7183
1.05	2.8577

1.10	3.0042
1.15	3.1582
1.20	3.3201
1.25	3.4903
1.30	3.6692

3. (a) Tabulate $y = x^3$ for x = 2, 3, 4 and 5, and use the successive approximation method to calculate the cube root of 12 correct to three decimal places.

(b) Tabulate $y = \sin(x)$ for $x = 0, \frac{\pi}{2}$, and π, and use cubic spline method to calculate the value of $y\left(\frac{\pi}{4}\right)$.

Unit II

4. (a) From the following data find x, correct to two decimal places for which y is maximum and find this value of y :

x	y
1.2	0.9320
1.3	0.9636

1.4	0.9855
1.5	0.9975
1.6	0.9996

(b) Use Romberg's method to compute $\int_0^1 \frac{1}{1+x^2} dx$ taking $h = 0.2$.

5. (a) Use Euler-Maclaurin's formula to compute $\int_0^1 \frac{1}{1+x} dx$ taking $h = 0.25$.

(b) Evaluate the following double integral $\int_{-2}^{2} \int_0^4 (x^2 + y^2) dx\, dy$ by using Widdle's rule.

Unit III

6. (a) Use Jacobi method to solve the following system of equations :

$$2x_1 + x_2 - x_3 = 1$$
$$5x_1 + 2x_2 + 2x_3 = -4$$
$$3x_1 + x_2 + x_3 = 5$$

(b) Use Gauss-Seidel method to solve the following system of equations :

$$3x_1 - x_2 + 2x_3 = 12$$

$$x_1 + 2x_2 + 3x_3 = 11$$

$$2x_1 - 2x_2 - x_3 = 2$$

7. (a) Find eigen values and corresponding eigen vectors of the following system :

$$\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$$

(b) Find the first two iterations of the SOR method with the relaxation parameter $\lambda = 1.1$ for the following linear systems, using $x^{(0)} = 0$:

$$3x_1 - x_2 + x_3 = 1$$

$$3x_1 + 6x_2 + 2x_3 = 0$$

$$3x_1 + 3x_2 + 7x_3 = 4$$

Unit IV

8. (a) Use Runge-Kutta method of fourth order to solve $\frac{dy}{dx} = 1 + y^2$ with $y(0) = 0$ on $[0, 1]$ with $h = 0.1$.

(b) Transform the second-order initial-value problem :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \text{ with } y(0) = -0.4,$$

$y'(0) = -0.6$ on $[0, 1]$.

into a system of first order initial-value problems and use the Runge-Kutta method with $h = 0.1$ to approximate the solution.

9. (a) Use the linear Shooting method to approximate the solution to the following boundary-value problems :

$$\frac{d^2y}{dx^2} = -3\frac{dy}{dx} + 2y + 2x + 3, \text{ for } 0 \leq x \leq 1$$

with $y(0) = 2$ and $y(1) = 1$; use $h = 0.1$.

(b) The differential equation $\frac{dy}{dx} = x^2 + y^2 - 2$ satisfies the following data :

x	y
-0.1	1.09
0	1.00
0.1	0.89
0.2	0.7605

Use Milne's method to obtain the value of $y(0.3)$.