Subject Code—11732

Dual Degree B. Sc. (Hons.) (Mathematics)-M.Sc. (Mathematics) EXAMINATION

(Batch 2016 Onwards)

(Third Semester)

VECTOR CALCULUS

BML-304

Time : 3 *Hours* *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Find the value of λ so that the following vectors are coplanar : $\vec{a} = 2\hat{i} - 7\hat{j} + \lambda\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - 5\hat{j} + 2\hat{k}$.

(b) Find a unit tangent vector to any point on the curve $x = a\cos t$, $y = a\sin t$, and $z = bt$.

(c) Prove that :

$$\nabla\phi . d\vec{r} = d\phi.$$

(d) Show that the vector function :

$$(x+3y)\hat{i} + (y-3z)\hat{j} + (x-2z)\hat{k}$$

is solenoidal.

(e) Prove that $\hat{e}_1 = h_2 h_3 \nabla u \times \nabla w$, with similar results for $\hat{e}_2$ and $\hat{e}_3$ where v, u, w are orthogonal co-ordinates.

(f) Find unit tangent vectors in cylindrical co-ordinate system.

(g) Using Gauss's Divergence theorem, Evaluate $\iint_S (xdydz + ydzdx + zdxdy)$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

Unit I

2. (a) Show that :

$$[\vec{a}\times\vec{p}\ \ \vec{b}\times\vec{q}\ \ \vec{c}\times\vec{r}]+[\vec{a}\times\vec{q}\ \ \vec{b}\times\vec{r}\ \ \vec{c}\times\vec{p}]$$

$$+[\vec{a}\times\vec{r}\ \ \vec{b}\times\vec{p}\ \ \vec{c}\times\vec{q}]=0.$$

(b) If $\vec{a}=\sin\theta\hat{i}+\cos\theta\hat{j}+\theta\hat{k},$

$$\vec{b}=\cos\theta\hat{i}-\sin\theta\hat{j}-3\hat{k}$$

and $\vec{c}=2\hat{i}+3\hat{j}-3\hat{k},$

find $\dfrac{d}{d\theta}\{\vec{a}\times(\vec{b}\times\vec{c})\}$ at $\theta=\dfrac{\pi}{2}.$

3. (a) If $\vec{r}$ is any vector, then show that :

$$\vec{r}=(\vec{r}.\vec{a}')\vec{a}+(\vec{r}.\vec{b}')\vec{b}+(\vec{r}.\vec{c}')\vec{c}.$$

(b) A particle moves along the curves $x=3t^2$, $y=t^2-2t$, $z=t^3$. Find its velocity and acceleration at $t=1$ in the direction of vector $\vec{a}=\hat{i}+\hat{j}-\hat{k}.$

Unit II

4. (a) Find the directional derivative of $\phi = 3y^2 + yz^3$ at the point (2, –1, 1) in the direction of normal to the surface $x \log z - y^2 + 4 = 0$ at point (–1, 2, 1).

   (b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$, then prove that :

$$\operatorname{div}(r^n \vec{r}) = (n+3)r^n.$$

5. (a) Prove that :

$$\operatorname{div}(\vec{f} \times \vec{g}) = \vec{g}.(\operatorname{curl} \vec{f}) - \vec{f}.(\operatorname{curl} \vec{g}).$$

   (b) Find $\vec{f} \times (\nabla \times \vec{g})$ at the point (1, –1, 2) if

$$\vec{f} = xz^2\hat{i} + 2y\hat{j} - 3xz\hat{k},$$

$$\vec{g} = 3xz\hat{i} + 2yz\hat{j} - z^2\hat{k}.$$

Unit III

6. (a) In orthogonal curvilinear co-ordinate system, find curl of a vector function $\vec{f}$.

(b) If (r, θ, ϕ) are spherical co-ordinates, then show that :

$$\nabla\left(\frac{1}{r}\right) = \nabla \times (\cos\theta \nabla \phi).$$

7. (a) Express the velocity $\vec{v}$ and acceleration $\vec{a}$ of a particle in cylindrical co-ordinates.

(b) Transform the following function from spherical to cartesian system :

$$\vec{f} = 3ar^2 \sin\theta \cos\phi \hat{e}_r + 2a^2 r \cos\theta \sin\phi e_\theta + r^3 e_\phi.$$

Unit IV

8. (a) Find the total work done in moving on a particle in a force field given by $\vec{f} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.

(b) Evaluate $\iint_S \phi \hat{n}\, dS$, where $\phi = \frac{3}{8}xyz$ and S is the surface of cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ to $z = 5$.

9. (a) State and prove Green's theorem.

(b) Verify Stokes' theorem for $\vec{f} = y\hat{i} + z\hat{j} + x\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.