Exam Code : M-26

Roll No. ....

Subject Code—30606

M.Sc. (UTD and Affiliated Colleges) (Under NEP 2020) EXAMINATION

(Second Semester)

(Batch 2025 Onwards)

MATHEMATICS

U25MAT212T

Advanced Numerical Methods

Time : 3 Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

P.T.O.

(Compulsory Question)

1. (a) Prove that $E^{1/2} = 1 + \frac{\delta}{2} + \frac{\delta^2}{8} + \ldots$

   (b) In which situations, numerical methods are used to find the root of nonlinear equation ?

   (c) Define numerical differentiation.

   (d) Define double interpolation.

   (e) Define different types of boundary value problems.

   (f) Define multistep methods to find the solution of differential equation.

   (g) Define the order of convergence of an iterative method.

Unit I

2. (a) Explain the method of successive approximation used for equally spaced data in inverse interpolation.

   (b) Using Aitken's method find y when $x = 2.0$ for the data $(1,-3),(3,9),(4,30)$ and $(6,132)$.

3. (a) Fit the natural cubic spline for a given set of data points (x_i, y_i), $i = 0,1,...N$ when the values of first derivative values at the end points are given as end conditions.

(b) Find the cube root of 21 by fitting a natural cubic spline to the data :

X :	0	1	8	27
$f(x)$:	0	1	2	3

Unit II

4. (a) From the following data, find the value of x for which y is maximum and find the value of y :

x	1	2	3	4	5
y	7	15	21	19	3

(b) Evaluate using $\int_1^2 \int_1^2 \frac{dx\, dy}{x+y}$ Trapezoidal rule.

P.T.O.

5. Derive Euler Maclaurin formula to solve $\int_{x_0}^{x_n} f(x)\, dx$ and use it find $\sum_{i=1}^{n} i^2$

Unit III

6. Explain the method to find the complex root of $f(z) = 0$. Hence find the complex roots of the equation $z^3+1=0$ correct upto 3 decimal places with intial approximation (0.25, 0.25).

7. (a) Using power method, find the largest eigenvalue and eigen vector of the matrix

$$\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{bmatrix}.$$

(b) Explain the SOR method to find the solution of linear system of equations with the value of accelerating factor.

Unit IV

8. (a) Derive Runga-Kutta formula of fourth order to solve the initial value problem

$$\frac{dy}{dx} = f(x, y); y(x_0) = y_0.$$

(b) Solve the boundary value problem by Numerov method

$$y'' = (1 + x^2) y$$

$$y(0) = 1, \qquad y(1) = 0 \qquad x \in [0,1]$$

with stepsize 0.2.

9. (a) Derive Milne predictor corrector formulas to solve the intial value problem

$$\frac{dy}{dx} = f(x, y); y(x_0) = y_0$$

(b) Solve using $\frac{dy}{dx} = x - y^2$ Milne predictor corrector formulas

given that $y(0)=0.000$, $y(0.2)=0.02$, $y(0.4)=0.0795$, $y(0.6)=0.1762$ for $y(0.8)$.

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