Roll No. ..... Exam Code : D-25

Subject Code—26118

M. Sc. (NEP 2020) EXAMINATION

(Batch 2025 Onwards)

(First Semester)

MATHEMATICS

U25MAT104T

Complex Analysis

Time : 3 Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **9** is compulsory. All questions carry equal marks.

Unit I

1. (a) To show that $f(z) = |z|$ is nowhere differentiable. 7

(b) Consider the map $w = \frac{1}{z}$ and determine the region R′ in W-plane of the infinite strip R bounded by $\frac{1}{4} < y < \frac{1}{2}$. 7

2. (a) State and Prove Cauchy-Goursat theorem. 10

(b) Verify Cauchy's theorem for the function $f(z) = z^3 - iz^2 - 5z + 2i$ if the path is circle given by $|z - 1| = 2$. 4

Unit II

3. (a) Find the value of :

$$\int_C \frac{(\sin z)^6}{\left(z - \frac{\pi}{6}\right)^3} dz$$

if C is circle $|z| = 1$. 7

(b) State and prove Morera's theorem. 7

4. (a) State and Prove Schwarz Lemma. 7

(b) State and Prove Poisson's Integral formula. 7

Unit III

5. (a) Obtain the Taylor's and Laurent's series which represents the function $\frac{z^2-1}{(z+2)(z+3)}$ in the regions :

(i) $|z| < 2$

(ii) $2 < |z| < 3$

(iii) $|z| > 3$

(b) State and Prove Rouche's theorem. 7

6. (a) To find the residue of :

$$f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$$

at $z = 1, 2, 3$. 7

(b) Prove that : 7

$$\int_0^{2\pi} \frac{\cos 2\theta \, d\theta}{5+4\cos\theta} = \frac{\pi}{6}$$

P.T.O.

Unit IV

7. (a) To show that the resultant (or product) of two bilinear transformations is a bilinear transformations. 7

(b) If the mapping $w = f(z)$ is conformal, then show that $f(z)$ is an analytic function of z. 7

8. (a) Find all the Mobius transformations which transformation the half plane $I(Z) \geq 0$ into circles $| W | \leq 1$. 7

(b) Explain the mapping $W = \log Z$ in detail with example. 7

Compulsory Question

9. (a) Define Branch point and Branch cut.

(b) Explain multiply connected domains with example.

(c) Define Liouville's theorem.

(d) Evaluate $\int_C \frac{e^z}{z-2} dz$ where C is $|z| = 1$.

(e) Define isolated, Removable and pole singularities with example.

(f) Find zeros and poles of $\left(\frac{z+1}{z^2+1}\right)^2$.

(g) Explain in brief Branches of many valued function of arg z. **2×7=14**