Roll No. .... Exam Code : S-21

Subject Code—11204

M. Sc. EXAMINATION

(Main & Re-appear for Batch 2019 Onwards)

(Second Semester)

MATHEMATICS

MAL-525

Complex Analysis—II

Time : 3 *Hours* *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

1. (a) Construct an entire function with simple zeros at the points 0, –1, –2, –3, ..., $-n$.

(b) Find the order of the function e^{z^λ}, λ being +ve integer.

(c) Define the germ of an analytic function.

(d) Show that :

$$\Gamma(z+1) = z\Gamma(z), z \neq 0, -1, -2, \ldots\ldots$$

(e) Define Landau's constant.

(f) State Subharmonic and Superharmonic function.

(g) State Bieberbach's conjecture. 14

Unit I

2. (a) State and prove Hurwitz's theorem.

(b) Show that the series :

$$\frac{1}{2} + \frac{z}{4} + \frac{z^2}{8} + \frac{z^3}{16} + \ldots.$$

and :

$$\frac{1}{2-i} + \frac{z-i}{(2-i)^2} + \frac{(z-i)^2}{(2-i)^3} + \ldots$$

are analytic continuation of each other.

7,7

3. State and prove Monodromy theorem. 14

Unit II

4. (a) State and prove Borel theorem.

(b) State and prove Hadamard three circle theorem. 7,7

5. State and prove Weierstrass factorization theorem. 14

Unit III

6. (a) Let G be a simply connected region and suppose that f is an analytic function on G that does not assume value 0 or 1. Let g be an analytic function on G such that $f(z) = -\exp\left(i\pi \cosh\left(2g(z)\right)\right)$ for z in G, then $g(G)$ contains no disc of radius 1.

(b) State and prove Montel-Caratheodory theorem. 7,7

7. State and prove Schottky's theorem. 14

Unit IV

8. (a) State and prove Poisson-Jenson formula.

(b) Let G and Ω be regions such that there is a one-one analytic function f of G onto Ω : let $a \in$ G and $\alpha = f(a)$. If g_a and γ_α are the Green's functions for G and Ω with singularities 'a' and 'α' respectively, then $g_a(z) = \gamma_\alpha(f(z))$. 7,7

9. State and prove Reimann mapping theorem. 14