Roll No. 2200606/0047

Exam Code : M-23

Subject Code—12005

M.Sc. EXAMINATION

(Batch 2016 Onwards)

(Second Semester)

MATHEMATICS

MAL-525

Complex Analysis—II

Time : 3 Hours *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory.

1. (a) Define normal and equicontinuous set of continuous functions.

(b) Define function element and direct analytic continuation.

(c) State Hadamard's three circle theorem.

(d) Find the order of sin z.

(e) State Schottky's theorem.

(f) State Poisson Jensen's formula.

(g) Define Riemann zeta function. 2×7=14

Unit I

2. (a) State and prove Hurwitz's theorem. 7

(b) Describe analytic continuation along a curve and show that it is unique. 7

3. (a) Explain, how it is possible to continue analytically the function,

$$f(z) = 1 + z + z^2 + \ldots + z^n + \ldots$$

outside the circle of convergence of the power series. 7

(b) Let $\gamma : [0, 1] \rightarrow C$ be a path from a to b and let $\{(f_t, D_t; 0 \leq t \leq 1)\}$ and $\{(g_t, B_t; 0 \leq t \leq 1)\}$ be analytic continuations along γ such that $[f_0]_a = [g_0]_a$. Then prove that $[f_1]_b = [g_1]_b$. 7

Unit II

4. (a) State and prove Jensen's formula. 7

(b) Let ρ be the order of an integral function $f(z)$, then show that :

$$\rho = \lim_{r\to\infty} \sup \frac{\log\log M(r)}{\log r},$$

where $M(r) = \max|f(z)|$ on $|z| = r$. 7

5. (a) Show that the order of a canonical product is equal to the exponent of convergence of its zeros. 7

(b) State and prove Hadamard's factorization theorem. 7

Unit III

6. State and prove Bloch's theorem. 14

7. (a) If f is an entire function that omits two values, then f is a constant (Little Picard Theorem.) 7

(b) State the prove Montel-Caratheodory Theorem. 7

Unit IV

8. (a) Let G be a bounded Dirichlet region. Then show that for each $a \in G$ there is a Green's function on G with singularity at a. 7

(b) Let G and Ω be regions such that there is a one-one analytic function f of G onto Ω; let $a \in G$ and $\alpha = f(a)$. If g_a and γ_α are the Green's function for G and Ω with singularities a and α respectively, then show that $g_a(z) = \gamma_\alpha(f(z))$. 7

9. (a) Derive the Riemann's functional equation : 7

$$\zeta(1-z) = 2^{1-z}\pi^{-z}\cos\frac{1}{2}\pi z\Gamma(z)\zeta(z)$$

(b) State and prove Harnack's inequality. 7