# Subject Code—18005

## M. Sc. EXAMINATION

(Batch 2020 Onwards)

(Second Semester)

MATHEMATICS

MAL-525

Complex Analysis–II

*Time* : 3 *Hours* *Maximum Marks* : 70

**Note** : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

**1.** (a) Show that there cannot be more than one analytic continuation of a function $f(z)$ into the same domain.

(b) Define Analytic Continuation along a path.

(c) State Weierstrass' Factorization Theorem.

(d) Find the order of polynomial :

$$P(z) = a_0 + a_1 z + \dots + a_n z^n,\ a_n \neq 0$$

(e) State Schottky's Theorem.

(f) State Bieberbach's conjecture and the 1/4 theorem.

(g) Define Green's function. **7×2=14**

## Unit I

2. Let $\{f_n\}$ be a sequence of function in M(G) and $f_n \to f$ in C(G, $C_\infty$), then either $f$ is meromorphic or $f \equiv \infty$. If each $f_n$ is analytic, then either $f$ is analytic or $f \equiv \infty$. **14**

3. State and prove Montel's Theorem. **14**

## Unit II

4. (a) State and prove Hadamard Factorization Theorem. **7**

(b) State and prove Jensen's formula. **7**



5. (a) If $f, f_1, f_2, f_3, \dots : X \to C$, where X is a set such that $f_n(x) \to f(x)$ uniformly for $x \in X$. If there is a constant '$a$' such that :

$$\operatorname{Re} f(x) \leq a \ \forall\, x \in X,$$

then $\exp f_n(x) \to \exp f(x)$ uniformly for $x \in X$. **7**

(b) Let $|a_n| = r_n$ $(0 < r_1 \leq r_2 \leq \dots \leq r_n \leq \dots,$ $r_n \to \infty)$, then the exponent of convergence is given by

$$\frac{1}{\rho_c} = \lim_{n \to \infty} \frac{\log r_n}{\log n}.$$ **7**

## Unit III

6. (a) Let G be a simply connected region and suppose that $f$ is an analytic function on G that does not assume the value 0 or 1. Then there is an analytic function $g$ on G such that $f(z) = -\exp(i\pi \cosh(2g(z)))$ for $z$ in G. **7**

(b) State and prove Little Picard Theorem. **7**

7. (a) Suppose $g(z)$ is analytic on $B(0; R)$, $g(0) = 0$, $|g'(0)| = \mu > 0$ and $|g(z)| \leq M$ for all $z$, then :

$$g\,(B(0;\,R)) \supset B\left(0; \frac{R^2\mu^2}{6M}\right).$$ 7

(b) State and prove Montel-Caratheodory Theorem. 7

## Unit IV

8. (a) Let G and $\Omega$ be regions such that there is a one-one analytic function $f$ of G onto Q; let $a \in G$ and $\alpha = f(a)$. If $g_a$ and $\gamma_\alpha$ are the Green's functions for G and $\Omega$ with singularities '$a$' and '$\alpha$' respectively, then $g_a(z) = \gamma_\alpha(f(z))$. 7

(b) Define Gamma function and derive Gauss's formula for it. Also show that : 7

$$\sin\,\pi z = \frac{\pi}{\Gamma(z)\,\Gamma(1-z)}; z \neq 0, \pm 1, \pm 2, .....$$

9. State and prove Harnack's Theorem. 14