Exam Code : M-23

Subject Code—12034

# B. Sc. (Hons.) EXAMINATION

(Batch 2016 Onwards)

(Fourth Semester)

MATHEMATICS

BML-403

## Elementary Partial Differential Equations

*Time* : 3 *Hours* *Maximum Marks* : 70

**Note** : Attempt *Five* questions in all. The question paper will consist of four sections. Q. No. 1 contains seven short answer type questions without any internal choice and are compulsory. Each of four Sections (I-IV) will contain two questions and the students are required to attempt *one* que[illegible] each Section. All questions c[illegible] marks.

1. (i) Define order and degree of a partial differential equation.

(ii) Define Lagrange's equation with suitable example.

(iii) Define complete solutions and singular solutions.

(iv) Define linear homogeneous partial differential of order $n$ with a suitable example.

(v) Define hyperbolic partial differential equation of second order in two independent variables.

(vi) State existence theorem for initial conditions of the first kind in linear hyperbolic equations.

(vii) Define characteristic equation.

## Section I

2. (a) Equation of any cone with vertex at P$(a, b, c)$ is of the form :

$$f\left(\frac{x-a}{z-c}, \frac{y-b}{z-c}\right)=0$$

Find differential equation of cone.

(b) Find partial differential equation from the following equation :

$$f\left(x^2 + y^2 + z^2, z^2 - 2xy\right) = 0$$

**3.** (a) Solve :

$$z(x+y)p + z(x-y)q = x^2 + y^2$$

(b) Solve :

$$p = 2q^2 + 1$$

## Section II

**4.** (a) Solve :

$$\left(\frac{\partial^3}{\partial x^3} - 7\frac{\partial^3}{\partial x \partial y^2} - 6\frac{\partial^3}{\partial y^3}\right) z = \cos(x + 2y)$$

(b) Solve :

$$\left(x^2 D^2 - 2xyDD' - 3y^2 D'^2 + xD - 3yD'\right) z = x^2 y \cos\left(\log x^2\right)$$

where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.

5. (a) Solve :

$$\left(D^2 - 4D'^2\right)z = 4\left(\frac{x}{y^2} - \frac{y}{x^2}\right)$$

where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.

(b) Solve :

$$\left(3DD' - 2D'^2 - 3D'\right)z = 3\cos(3x - 2y)$$

## Section III

6. (a) Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

(b) Reduce $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$ into corresponding canonical form.

7. Solve by using Monge's method $(r - s)x = (t - s)y$, notations have their usual meaning.

## Section IV

8. (a) Find the real characteristic of :

$$\left(\sin^2 x\right)\frac{\partial^2 z}{\partial x^2}+(2\cos x)\frac{\partial^2 z}{\partial x \partial y}-\frac{\partial^2 z}{\partial y^2}=0$$

(b) Solve :

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$$

with the conditions $u(0, t)=u(l, t)=0$ $u(x, 0)=x(l-x)$.

9. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ in $xy$ plane, $0 \leq x \leq a$, $0 \leq y \leq b$ with the conditions :

$$u(0, y)=0, u(a, y)=0, 0 \leq y \leq b$$