Exam Code : D-23

Subject Code—15057

Dual Degree B. Sc. (Hons.) Mathematics-M. Sc. Mathematics EXAMINATION

(Batch 2017 Onwards)

(Fifth Semester)

GROUPS AND RINGS

BML-502

Time : 3 *Hours* *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. 9 is compulsory. All questions carry equal marks.

Section I

1. (a) Show that the set $G = \{a + b\sqrt{2}; a, b \in Q\}$ is an abelian group w.r.t. addition. 7

(b) Let $G = \{0, 1, 2, 3, 4, 5\}$. Find the orders of elements of the group G under the binary operation addition modulo 6. 7

2. (a) Every subgroup of a cyclic group is cyclic. Prove. 7

(b) State and prove Lagrange's theorem. 7

Section II

3. (a) Show that every homomorphic image of a group G is isomorphic to some quotient group of G. 7

(b) Every inner automorphism of a group is automorphism of that group. 7

4. (a) If f is an automorphism of a group G and $a \in G$ be an element, then show that $f(N(a)) = N(f(a))$. 7

(b) Write all elements of symmetric group S_3 as product of disjoint cycles. 7

Section III

5. (a) Every finite non-zero integral domain is a field. Prove. 7

   (b) The order of each non-zero element of an integral domain is same. Prove. 7

6. (a) If S_1, S_2 are two ideals of a ring R, then their product S_1S_2 is an ideal of R. 7

   (b) Every Homomorphic image of a ring R is isomorphic to some quotient ring. 7

Section IV

7. (a) The integral domain $\langle Z, +, \cdot \rangle$ of integers is a Euclidean domain. 7

   (b) Show that every non-zero prime ideal of a principal ideal domain is maximal. 7

8. (a) If F is a field, then $F[x]$ is a principal ideal ring. 7

   (b) In a VFD every pair of non-zero elements have a g.c.d. and l.c.m. 7

Compulsory Question

9\. (a) Give an example that union of two subgroups in not necessarily a subgroup.

(b) Find the right cosets of the sub-group $\{1, -1\}$ of the group $\{1, -1, i, -i\}$ w.r.t. usual multiplication.

(c) Define equality of permutations with example.

(d) If $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$, show that AB = BA.

(e) Prove that the set $\{0, 1, 2, 3, 4, 5\}$ with addition modulo 6 and multiplication modulo 6 as composition is a ring with zero divisors.

(f) Define embedded ring and set of quotients of a ring.

(g) State proper and improper divisors.

$2 \times 7 = 14$