Roll No. .... **Exam Code : M-24**

Subject Code—18060

Dual Degree B.Sc. (Hons.) EXAMINATION

(Batch 2017 Onwards)

(Sixth Semester)

MATHEMATICS

BML-602

Linear Algebra

Time : 3 Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all. The question paper consists of four Units. Question No. **1** is compulsory. Attempt any *one* question out of two from each Unit. All questions carry equal marks.

(Compulsory Question)

1. (i) Whether Q (set of rationals) is a vector space over R (reals) under usual addition

P.T.O.

and usual multiplication operations ? Justify your answer with an example.

(ii) Find dimension of vector space $Q(\sqrt{2})$ over Q.

(iii) State Sylvester's law on linear transformations.

(iv) Show that linear transformation $T : R^3 \to R^3$ defined by $T(x, y, z) = (x + z, x - z, y) = 14$ is non-singular.

(v) State Cauchy Schwarz inequality and triangle inequality.

(vi) Define a normed linear space.

(vii) Define Adjoint operator and Self-adjoint operator on an inner product space.

2×7=14

Unit I

2. (a) Prove that the set of all diagonal matrices over R (set of reals), is a vector space with respect to matrix addition and scalar multiplication. 7

(b) If a subspace W of R^4 is generated by the vectors $(3, 8, -3, -5)$, $(1, -2, 5, -3)$ and $(2, 3, 1, -4)$; find a basis and dimension of W. Also, extend this basis to get a basis of R^4. 7

3\. (a) Find a complement of a subspace W generated by $(1, 0, 1)$ and $(1, 2, 3)$ in $V = R^3(R)$. 7

(b) If W is a subspace of a vector space V(F) and $u, u' \in V$, then $W + u = W + u'$ iff $u - u' \in W$. 7

Unit II

4\. (a) Show that the map $T : R^3 \to R^2$ defined by $T(x, y, z) = (|x|, y, z)$ is not a linear transformation. 7

(b) Let $u_1 = (1, 1)$, $u_2 = (0, 1)$ be a basis of R^2. Let $T : R^2 \to R$ be the linear transformation for which $T(u_1) = 3$ and $T(u_2) = -2$. Find the linear transformation. 7

5. (a) Let $T : U \to V$ be a linear transformation, then prove : **7**

$$\frac{U}{Ker\,T} \cong T(U).$$

(b) Find a linear transformation $T : R^3 \to R^4$ whose range is generated by $(1, 2, 0, -4)$ and $(2, 0, -1, -3)$. **7**

Unit III

6. (a) Let $T : R^3 \to R^3$ be a linear operator defined by $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$. Show that T is invertible and find T^{-1}. **7**

(b) Find the matrix representing the transformation $T : R^3 \to R^4$ defined by $T(x, y, z) = (x + y + z, 2x + z, 2y - z, 6y)$ relative to the standard basis of R^3 and R^4. **7**

7. (a) For the linear opertor $T : R^3 \to R^3$, find the eigen values and the basis for the eigen space, when $T(x, y, z) = (2x + y, y - z, 2y + 4z)$. **7**

(b) Let T be a linear opertor on $R^2(R)$ defined by $T(x, y) = (x + y, x - y)$. Find the characteristic polynomial and minimal polynomial for T. 7

Unit IV

8. (a) Obtain an orthonormal basis with respect to standard inner product for the subspace of R^3 generated by (1, 1, 1), (1, –2, 1) and (1, 2, 3). 7

(b) Let V(F) be an inner product space. If $u, v \in V$ such that $|\langle u, v \rangle| = \|u\| . \|v\|$, then show that u and v are lineraly dependent. 7

9. (a) Let S be a subset of an inner product space V. Then show that $S^{\perp} = S^{\perp\perp\perp}$. 7

(b) Prove that every inner product space is a metric space. 7

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