Subject Code—18003

M. Sc. EXAMINATION

(Batch 2020 Onwards)

(Second Semester)

MATHEMATICS

MAL-523

Methods of Applied Mathematics

Time : 3 *Hours* *Maximum Marks* : 70

Note : Q. No. **1** is compulsory. Attempt *Five* questions in all, selecting *one* question from each Unit I-IV. All questions carry equal marks.

Compulsory Question

1. (a) Find Fourier transform of $f(t)=1$, $-a \le t \le a$.

(b) Find Fourier sine transform of $f(t)=\frac{1}{t}$; $t > 0$.

(c) Determine transformation from cylindrical to rectangular co-ordinates.

(d) A continuous random variable X has pdf :

$f(x)=3x^2, \qquad 0 \le x \le 1$ and

$P(X > b) = 0.05$

find value of 'b'.

(e) What do you mean by covariant components of a vector ?

(f) If X is a Poisson variate such that P(X = 1) = P(X = 2), find mean of distribution.

(g) Define F-statistic and write density function of F-distribution.

Unit I

2. (a) Find Fourier sine transform of

$$\frac{1}{t\left(t^2+a^2\right)}.$$

(b) Solve, using Fourier transforms :

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, \qquad x>0, t>0$$ Subject to

$u(x,0)=0$, $u_x(0,t)=-k$ and $u(x,t)$ is bounded.

3. (a) Find Fourier transform of

$$f(t)=\begin{cases} t, & |t|\le 1 \\ 0, & |t|>1 \end{cases}.$$

(b) Find solution of :

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=\sin wt; \text{ for } t>0$$

Unit II

4. (a) Obtain expression for gradient of a scalar point function in orthogonal curvilinear co-ordinates (u_1, u_2, u_3).

(b) Prove that cylindrical co-ordinate system, is orthogonal.

5. (a) Find the relation between the contravariant components of a given vector in two general curvilinear co-ordinate systems (u_1, u_2, u_3) and $(\bar{u}_1, \bar{u}_2, \bar{u}_3)$.

(b) Represent the vector $\vec{A} = 2y\hat{i} - z\hat{j} + x\hat{k}$ in spherical co-ordinates.

Unit III

6. (a) Find moment generating function of a r.v. X whose pdf is :

$$f(x) = \begin{cases} x; & 0 < x \le 1 \\ 2-x; & 1 \le x \le 2 \\ 0; & \text{elsewhere} \end{cases}$$

Also deduce values of mean and variance.

(b) Prove the following for Poisson distribution :

$$\mu_{r+1} = m\left(r\mu_{r-1} + \frac{d\mu_r}{dm}\right)$$

7. (a) Define geometrical distribution. Obtain its moment generating function and deduce value of mean and variance.

(b) What do you mean by exponential distribution ? Obtain mean deviation about mean and cumulants of this distribution.

Unit IV

8. (a) Prove that for a normal distribution :

$$\mu_{2n} = 1.3.5 \ldots\ldots\ldots (2n-1)\sigma^{2n}$$

(b) Define multiple correlation and obtain the relation :

$$\sigma_{1.23}^2 = \frac{\omega}{\omega_{11}}\sigma_1^2$$

where symbols have their usual meanings.

9. (a) Define t-statistic and obtain moments of t-distribution. Deduce value of variance.

(b) State and prove Central limit theorem.