

Exam Code : N-21

M.Sc. EXAMINATION

(Second Semester)

MAL-523

Time : 3 Hours

Maximum Marks : 70

$$\frac{1}{x} - x^{-\frac{1}{2}}$$

(Compulsory Question)

1. (a) Find the Fourier sine transform of $\frac{1}{t}$.

$$-\theta \int \frac{x e^{-\theta x}}{-\theta} dx$$

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P.T.O.

$$-\frac{d}{dx} \left[\frac{1}{2e^{-0.2x}} \right] = \left[\frac{1}{2} e^{-0.2x} \right] - \left[\frac{1}{2} e^{-0.2x} \right] \cdot (-0.2)$$

- (b) State and prove change of scale property on Fourier transforms.
- (c) Define contravariant and covariant components of a vector.
- (d) Define sample space and discrete random variable along with *one* example for each.
- (e) Define Uniform distribution and obtain its variance.
- (f) Show that the sum of two independent normal variates is also a normal variate.
- (g) Define F-Distribution.

Unit I

2. (a) Obtain Fourier cosine transform of :

$$f(t) = e^{-\frac{t^2}{2}}.$$

- (b) Prove that Fourier transform of the product of two functions is the convolution of their Fourier transforms.

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$$a. \frac{1}{a} e^{-\frac{t^2}{4a^2}} [e^{-\frac{s^2}{4}}]_2 = \frac{1}{a} e^{-\frac{1}{4}(\frac{s^2}{a^2})}$$

$$= \frac{1}{a} \cdot e^{-\frac{s^2}{4a^2}}$$

$$= \frac{1}{\sqrt{2a}} e^{-\frac{s^2}{2a}}$$

3. (a) Solve, using Fourier transforms :

$$\frac{\partial V}{\partial t} = 4 \frac{\partial^2 V}{\partial x^2}, \text{ subject to } V(0, t) = 0;$$

$V(x, 0) = e^{-2x}$ and $V(x, t)$ is bounded for $x > 0, t > 0$.

(b) Find Fourier transform of $f(t) = e^{-|t|}$.

Unit II

4. (a) Prove that spherical co-ordinate system is orthogonal.

(b) Obtain expression for velocity and acceleration of a particle in cylindrical co-ordinates (r, ϕ, z) .

5. (a) Obtain expression for $\text{curl } \vec{F}$ in orthogonal curvilinear co-ordinates and deduce its expression in cylindrical co-ordinates.

(b) Derive the relation between the contravariant components of a vector in two co-ordinate systems (u_1, u_2, u_3) and $(\bar{u}_1, \bar{u}_2, \bar{u}_3)$.

$$n(n-1)(n-2)\dots(n-2-1)(n-2-1)$$

$$n!$$

$$n(n-1)(n-2)$$

$$(n-2)$$

$$n-1$$

$$n(n-1)$$

Unit III

6. (a) Find mean, variance and mean deviation for the probability function :

$$f(x) = \frac{1}{2}e^{-|x|}; -\infty < x < \infty$$

- (b) Define M.G.F. If X assumes the value "x" with probability :

$$P(X = x) = pq^{x-1}, x = 1, 2, 3, \dots$$

Find its M.G.F. and hence deduce the values of mean, variance and standard deviation.

7. (a) Derive Poisson distribution as a limiting case of Binomial distribution.
- (b) Define exponential distribution. Obtain mean deviation about mean for this distribution.

Unit IV

8. (a) Prove that for a normal distribution, the odd order moments are zero and even order moments are given by $\mu_{2n} = 1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)\sigma^{2n}$.

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$$e^{-mx} \cdot \frac{m^2}{2!} \cdot e^{-m}$$

$$1 + m + \frac{m^2}{2!} + \dots$$

