Roll No. 2200606 10047. Exam Code : M-23

Subject Code—12003

M. Sc. Mathematics EXAMINATION

(Batch 2016 Onwards)

(Second Semester)

MATHEMATICS

MAL-523

Methods of Applied Mathematics

Time : 3 Hours *Maximum Marks : 70*

Note : Q. No. **1** is compulsory. Attempt *Five* questions in all, selecting *one* question from each of Units I-IV and the compulsory question. All questions carry equal marks.

Compulsory Question

1. (a) Find Fourier cosine transform of e^{-at}, $a > 0$.

(b) State and prove shifting property on Fourier transforms.

(c) What do you mean by contravariant components of a vector ?

(d) Write the unit vectors $\hat{e}_r, \hat{e}_\theta, \hat{e}_\phi$ of a spherical co-ordinate system in terms of $\hat{i}, \hat{j}, \hat{k}$.

(e) Find mean and standard deviation for a random variable X with p.d.f. :

$$f(x) = \begin{cases} 2(x-1), & 1 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

(f) Define uniform distribution and obtain its moment generating function.

(g) State Central limit theorem.

Unit I

2. (a) Find the Fourier cosine transform of $f(t) = e^{-t^2}$.

(b) Solve, using Fourier transforms :

$$\frac{\partial V}{\partial t} = 2\frac{\partial^2 V}{\partial x^2}$$

$x > 0, t > 0$, $V(0, t) = 0$; $V(x, 0) = e^{-x}$, $x > 0$ and $V(x, t)$ is bounded.

3. (a) Define Fourier sine transform and evaluate it for the function :

$$f(t) = \frac{e^{-at}}{t}, a > 0$$

(b) State and prove convolution theorem on Fourier transforms.

Unit II

4. (a) Obtain expression for divergence of a vector point function in orthogonal curvilinear co-ordinates (u_1, u_2, u_3).

(b) Prove that cylindrical co-ordinate system is orthogonal.

5. (a) Represent the vector $\vec{F} = x\hat{i} - z\hat{j} + 2y\hat{k}$ in spherical co-ordinates (r, θ, ϕ).

(b) If (u_1, u_2, u_3) are general co-ordinates, show that $\frac{\partial \vec{r}}{\partial u_1}, \frac{\partial \vec{r}}{\partial u_2}, \frac{\partial \vec{r}}{\partial u_3}$ and $\nabla u_1, \nabla u_2, \nabla u_3$ are reciprocal system of vectors.

Unit III

6. (a) If density function for a random variable is :

$$f(x) = \frac{1}{a}\left[1 - \frac{|x-b|}{a}\right], |x-b| < a$$

find E (X) and variance of the distribution.

(b) Define a Binomial distribution and prove the following recurrence relation for this distribution :

$$\mu_{r+1} = pq\left[nr\mu_{r-1} + \frac{d\mu_r}{dp}\right]$$

7. (a) In a Poisson distribution, frequency corresponding to 3 successes is $\frac{2}{3}$ times the frequency curresponding to 4 successes. Obtain the mean and variance of the distribution.

(b) Show that all cumulants of Poisson distribution are equal.

(c) Define exponential distribution. Obtain its M.G.F. and mean.

Unit IV

8. (a) Prove that for a normal distribution, the standard deviation is the distance from the axis of symmetry to a point of inflexion.

(b) Define gamma distribution. Obtain M.G.F., mean and variance for this distribution.

9. (a) Define multiple and partial correlations. Show that :

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{\left(1-r_{13}^2\right)\left(1-r_{23}^2\right)}}$$

(b) Define t-distribution. Obtain its mean and variance. Also define F-distribution.