Exam Code : D-23

Subject Code—15049

Dual Degree B.Sc. (Hons.) Mathemtics-M.Sc. Mathematics (Third Semester) (Batch 2017 Onwards) EXAMINATION

NUMBER THEORY AND TRIGONOMETRY

BML-301

Time : 3 Hours

Maximum Marks : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. 1 is compulsory.

(Compulsory Question)

1. (i) If $a|b$ and $a|c$, then prove that $a|(bx + cy)$ for all integral values of x and y.

(ii) Find the remainder on dividing the sum $1! + 2! + 3! + 4! + ... + 30!$ by 12.

(iii) Show that $n^5 - n$ is divisible by 30.

(iv) Show that $\phi(1155)$ is multiple of 16.

(v) Define reduced residue system.

(vi) If $2\cos\alpha = x + \frac{1}{x}$, $2\cos\beta = y + \frac{1}{y}$, then calculate $x^m y^n + \frac{1}{x^m y^n}$.

(vii) Separate $\tan(x+iy)$ into real and imaginary parts. **7×2=14**

Section I

2. (a) Use canonical decomposition to find g.c.d and l.c.m. of 7200, 3132. 7

(b) Find the remainder when 4444^{4444} is divided by 9. 7

3. (a) Solve the congruence $342x \equiv 5 \pmod{13}$. 7

(b) Solve the following set of congruences : $2x \equiv 3 \pmod 5$, $4x \equiv 2 \pmod 6$ and $3x \equiv 2 \pmod 7$. 7

Section II

4. (a) Show that $\{2, 4, 6, \ldots\ldots, 2m\}$ is a complete residue system (mod m) if m is odd. 7

   (b) Find the highest power of 5 contained in 1000!. 7

5. (a) Solve the congruence $x^2 \equiv 5 \pmod{29}$. 7

   (b) Write all the quadratic residues of 7. 7

Section III

6. (a) Find by De Moivre's theorem, the roots of $x^9 - 1 = 0$ and point out which of these roots are also the roots of $x^3 = 1$. 7

   (b) Express $\sin^8 \theta$ in a series of cosines of multiples of θ. 7

7. (a) If α and β are imaginary cube roots of unity, then prove that : 7

$$\alpha . e^{\alpha x} + \beta e^{\beta x} = -e^{\frac{-x}{2}} \left[\sqrt{3} \sin \frac{\sqrt{3}}{2} x + \cos \frac{\sqrt{3}}{2} x \right].$$

(b) Prove that : 7

$$\cos(\alpha + i\beta) + i\sin(\alpha + i\beta) = e^{-\beta}(\cos\alpha + i\sin\alpha)$$

Section IV

8. (a) Prove that $(i^i)^i = \cos\theta - i\sin\theta$ where $\theta = (4n+1)\frac{\pi}{2}$. 7

(b) Prove that : 7

$$\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{3}{5} = \tan^{-1}\frac{27}{11}.$$

9. (a) With the help of Gregory's series, prove that : 7

$$\frac{\pi}{\sqrt{2}} = 2\sqrt{2}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\right)$$

(b) Find the sum of the series :

$\sin\alpha - \sin 2\alpha + \sin 3\alpha - \sin 4\alpha + \ldots\ldots$ up to n terms. 7