Roll No. .... **Exam Code : D-22**

Subject Code—11731

Dual Degree B. Sc. (Hons.) (Mathematics)-M.Sc. (Mathematics) EXAMINATION

(Batch 2016 Onwards)

(Third Semester)

ADVANCED CALCULUS

BML-303

Time : 3 Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Differentiate $\sqrt{1+x^2}$ w.r.t. x.

(b) State Taylor's theorem with Lagrange's form of remainder.

P.T.O.

(c) Prove that the function :

$f : A \to R,\ A \subset R^2$ defined by

$$f(x, y) = \begin{cases} x \sin \frac{1}{y}, & y \neq 0 \\ 0, & y = 0 \end{cases}$$

is continuous at (0, 0).

(d) Find the first order partial derivatives of e^{x^y}.

(e) State Schwarz's theorem.

(f) Find direction ratios of Binormal.

(g) If a curve lies on a sphere, show that ρ and σ are related by the relation :

$$\frac{d}{ds}(\sigma\rho') + \frac{\rho}{\sigma} = 0.$$

7×2=14

Unit I

2. (a) Show that $f(x) = \frac{1}{x^2}$ is not uniformly continuous on (0, 1). 7

(b) Verify Rolle's theorem for the function $f(x) = \frac{\sin x}{e^x}$ in $[0, n]$. 7

3. (a) Using Lagrange's mean value theorem find a point on the curve $y = \sqrt{x-2}$ defined in the interval [2, 3], where the tangent is parallel to the chord joining the end points of the curve. 7

(b) Evaluate the indeterminant form : 7

$$\lim_{x \to 0} x \log \tan x.$$

Unit II

4. (a) Show that $f(x, y) = \sqrt{|xy|}$ is continuous at the origin. 7

(b) If $u = \cos^{-1} \frac{x + y}{\sqrt{x} + \sqrt{y}}$, show that : 7

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0.$$

(b) Show that the radius R of the spherical curvature is given by $R^2 = \rho^4 \sigma^2 r'''^2 - \sigma^2$. 7

9\. (a) Find the involute of the twisted cubic $x = u,\ y = u^2,\ z = u^3$. 7

(b) Find the envelope of the plane $\frac{x}{a+u} + \frac{y}{b+u} + \frac{z}{c+u} = 1$, where u is the parameter. 7