Exam Code : M-23

Subject Code—12032

B.Sc. (Hons.) EXAMINATION

(Batch 2016 Onwards)

(Fourth Semester)

MATHEMATICS

BML-401

Solid Geometry

Time : 3 *Hours*

Maximum Marks : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. 1 is compulsory. All questions carry equal marks.

(Compulsory Question)

1. Answer any *seven* of the following : **7×2=14**

(a) Find the nature of conic $36x^2 + 24xy + 29y^2 = 180$.

(b) Define Confocal Parabolas.

(c) Find the equation of sphere, through the circle $x^2 + y^2 + z^2 = 1$, $x + 2y + 3z = 4$ and origin.

(d) Find the enveloping cone of sphere $x^2 + y^2 + z^2 + 2x - 2y = 2$ with vertex at (1, 1, 1).

(e) Define conicoid and prove that all plane section of central conicoid are conic.

(f) Find the equation of the plane which cuts the paraboloid $x^2 - 2y^2 = z$ in conic with centre at point $\left(2, \frac{3}{2}, 4\right)$.

(g) Prove that one conicoid confocal with a given conicoid, touches a plane.

(h) Determine the circular sections of the paraboloid $ax^2 + by^2 = 2cz$.

Section I

2. (a) Show that the equation

$$ax^2 + 24xy + 16y^2 - 2x + 14y + 1 = 0$$

represent a parabola. Also find its focus, vertex, axis and latus rectum. 7

(b) Prove that one and only one conic of a confocal system wire touch given straight line. 7

3. (a) Find the polar equation of conic with focus at pole. 7

(b) Find the condition that the line $\frac{e}{r} = a\cos\theta + b\sin\theta$, may be tangent to conic $\frac{e}{r} = 1 + e\cos\theta$. 7

Section II

4. (a) Obtain equation of the sphere, having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as a great circle. 7

(b) Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at the point P(1, –2, 1) and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$. 7

5. (a) Find the equation of right circular cone whose vertex is at origin, axes the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has vertical angle of 60°. 7

(b) Find the equation of right circular cylinder whose axes is $x = 2y = -z$ and radius 4. 7

Section III

6. (a) To prove that six normal can be drawn from a point (a, b, c) to the ellipsoid $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} + \frac{z^2}{\gamma^2} = 1$. 7

(b) Prove that the sum of the squares of the reciprocals of any *three* mutually perpendicular diameters of an ellipsoid is constant. 7

7. (a) Find the equation of the polar plane of (a, b, c) w.r.t., ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. 7

(b) Find the centre of conic given by equation $2x - 2y - 5z + 5 = 0$, $3x^2 + 2y^2 - 15z^2 = 4$. 7

Section IV

8. (a) Find the locus of point from which three mutually perpendicular lines can be drawn to the paraboloid $ax^2 + by^2 + 2z = 0$. 7

(b) Prove that the equation $7x^2 + y^2 + z^2 + 8zx + 16yz - 8xy + 2x + 4y - 10z - 14 = 0$ is a hyperboloid of revolution of one sheet. Find the required equation and equation of its principal axis. 7

P.T.O.

9. (a) Find the locus of point of intersection of perpendicular generators of the hyperboloid paraboloid. 7

(b) Show that locus of the point of intersection of three planes mutually at the right angles, each of which touches one of three given confocals on sphere. 7