Roll No. .....

D-22

Subject Code—11729

Dual Degree B. Sc. (Hons.)-M.Sc.

EXAMINATION

(Batch 2016 Onwards)

(Third Semester)

MATHEMATICS

BML-301

Number Theory and Trigonometry

Time : 3 *Hours* *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

1. (a) Prove that any two consecutive integers are co-prime.

   (b) Show that :

$$3^{91} \equiv 4 \pmod{23}$$

(c) Evaluate $\phi(462)$.

(d) Define Sigma function.

(e) Find the real and imaginary parts of $e^{5+i\frac{\pi}{2}}$.

(f) Prove that :

$$\cosh^{-1} x = -i\cos^{-1} x.$$

(g) Find the general values of i^i.

Section I

2. (a) If $2^{n+1} + 1$ is an odd prime, then show that n is a power of 2.

(b) Prove that there is no pair of integers x, y satisfying $x + y = 100$ and $(x, y) = 7$ simultaneously.

3. (a) Find all the solutions of the Diophantine equation $15x + 7y = 111$.

(b) Solve the congruence :

$$103x \equiv 57 \pmod{211}$$

Section II

4. (a) Show that $1^2, 2^2, 3^2, \ldots\ldots m^2$ is not CRS (mod m), if $m > 2$.

(b) Prove that product of two odd primes cannot be a perfect number.

5. (a) Let p be an odd prime and a be an integer $(p, a) = 1$, then either $a^{\frac{p-1}{2}} \equiv 1 (\text{mod } p)$.

(b) Show that 2 is a quadratic non-residue modulo 13.

Section III

6. (a) Find the general value of θ which satisfies :

$$(\cos\theta + i\sin\theta)(\cos 3\theta + i\sin 3\theta)\ldots\ldots\ldots$$
$$(\cos(2n-1)\theta + i\sin(2n-1)\theta) = 1.$$

(b) Find all the values of $\left(1+\sqrt{-3}\right)^{\frac{3}{4}}$ and show that their continued product is 8.

7. (a) If $\tan(A+iB)=x+iy$, where A, B, x, y are real numbers, prove that :

$$x^2+y^2-2y\cot 2B=-1.$$

(b) Let m and n be integers such that $(m, n) = 1$, evaluate :

$$\sum_{k=1}^{n}\left(e^{\frac{2\pi ki}{n}}\right)^m.$$

Section IV

8. (a) Separate the real and imaginary parts in $\log(-1+i)-\log(-1-i)$.

(b) Prove that :

$$\sin^{-1}(i)=n\pi+(-1)^n i\log\left(\sqrt{2}+1\right),\ 0<\theta<\pi.$$

9. (a) Prove that :

$$\tanh^{-1} z = \frac{1}{2}\cosh^{-1}\frac{1+z^2}{1-z^2}.$$

(b) Sum the series $\cos^3\alpha + \cos^3(\alpha+\beta) + \cos^3(\alpha+2\beta)$..... upto n terms.