Roll No. ....

Exam Code : D-25

Subject Code—26117

M.Sc. (Under NEP-2020) EXAMINATION

(Batch 2025 Onwards) (Re-appear)

(First Semester)

MATHEMATICS

U25MAT103T

Ordinary Differential Equation-I

Time : 3 Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

Compulsory Question

1. (a) Define an integral equation along with its types.

(b) Show that the function $f(t, y) = t^2|y|$ defined on the rectangle :

$$R = \{t, y\} : |t| \leq 1, |y| \leq 1\}$$

satisfies the Lipschitz condition.

(c) Explain the concept of equicontinuous family of functions.

(d) Define maximal internal of existence of solution of an IVP.

(e) Prove that a Pfaffian differential equation in two variables always possesses a solution.

(f) Define oscillatory differential equations and provide an example.

(g) Describe Sturm-Liouville Boundary value problem. Provide an example. **7×2=14**

Unit I

2. (a) State and prove Cauchy Euler construction theorem. 7

(b) Define on IVP. Prove that solution of an IVP is equivalent to that of a volterra integral equation. 7

3. State and prove Picard-Lindel of theorem for local existence and uniqueness of solutions. 14

Unit II

4. (a) Obtain a power series solution in power of x of the IVP : 7

$$\frac{dy}{dx} = x^2 + 2y^2, \; y(0) = 4$$

by Taylor series method.

(b) Apply the Runge-Kutta method to solve the IVP : 7

$$\frac{dy}{dx} = 2x + y, \; y(0) = 1$$

at $x = 0.2$ and $x = 0.4$ using $h = 0.4$. 7

5. (a) Explain the behaviour of the solution of an IVP depending upon the function. 7

(b) State and prove the extension theorem for the solution of an IVP. 7

Unit III

6. (a) A necessary and sufficient condition that there exists between two functions $u(x, y)$ and $v(x, y)$ a relation $F(u, v) = 0$, not in involving x or y explicitly is that : 7

$$\frac{\partial(u,v)}{\partial(x,y)} = 0$$

(b) Verify that the equation :

$$2yzdx - 2xzdy - (x^2 - y^2)(z - 1)dz = 0$$

is integrable and find its primitive. 7

7. (a) State and prove Cromwell's inequality. 7

(b) Let $f(t, x)$ satisfies a Lipschitz condition for $t \geq a$. If the function $u(t)$ satisfies $x'(t) \leq f(t, x)$ for $t \geq a$ and $v(t)$ is a solution of $x'(t) = f(t, x)$, $t \geq a$ such that $u(a) = v(a) = c$, prove that $u(t) \leq v(t)$, $t \geq a$. 7

Unit IV

8. (a) Define Riccati equations and obtain a relationship between Riccati equation and linear differential equation of record order. 7

(b) Let $f(t)$ be a solution of differential equation :

$$[p(t)\ u'(t)]' + q(t)\ u(t) = 0$$

such that $f(t)$ has an infinite number of zero on $[a, b]$. Show that $f \cong 0$ in $[a, b]$. 7

9. (a) State and prove Sturm separation theorem. Provide an example. 7

(b) Prove that eigen values of sturm Liouville boundary value problem are real and discrete. 7