Roll No. 2200 6061 0047.

Exam Code : M-23

Subject Code—12004

M.Sc. EXAMINATION

(Batch 2016 Onwards)

(Second Semester)

MATHEMATICS

MAL-524

Ordinary Differential Equations-II

Time : 3 Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Section V is compulsory. All questions carry equal marks.

Section I

1. (a) Let $\phi_1, \phi_2, \ldots\ldots \phi_n$ be n vector function solutions of the homogeneous linear differential equation :

$$\frac{dx}{dt} = A(t)x$$

on real internal $[a, b]$. Show that $\phi_1, \phi_2, \ldots\ldots \phi_n$ are linearly independent if and only if $W(\phi_1, \phi_2, \ldots\ldots \phi_n)(t) \neq 0 \ \forall \ t \in [a, b]$. 7

(b) Show that $\Phi(t) = \begin{pmatrix} e^{5t} & e^{3t} \\ e^{5t} & 3e^{3t} \end{pmatrix}$ is a fundamental matrix of the linear system

$$\frac{dx}{dt} = \begin{pmatrix} 6 & -1 \\ 3 & 2 \end{pmatrix} x$$

Also find unique solution ϕ so that $\phi(0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$. 7

2. (a) State and prove Abel-Liouville formula. 7

(b) Show that $\det(\Phi(t)) \neq 0$, for $t \in [a, b]$ is necessary and sufficient for a solution matrix Φ of matrix differential equation $X' = A(t)X \ (t \in I)$ to be a fundamental matrix. 7

Section II

3. (a) Write a note on method of variation of constant for a non-homogeneous system. 7

(b) Show that if, for n times continuously differentiable function $\phi_1, \phi_2, \ldots\ldots \phi_n$, Wronskian $W(\phi_1, \phi_2, \ldots\ldots \phi_n)(t) \neq 0$ then $\exists$ a unique homogeneous differential equation of order n for which these functions form a fundamental set. 7

4. (a) Find the unique solution ϕ of the non-homogeneous system :

$$\frac{dx}{dt} = \begin{pmatrix} -10 & 6 \\ -12 & 7 \end{pmatrix} x + \begin{pmatrix} 10e^{-3t} \\ 18e^{-3t} \end{pmatrix}$$

that satisfies the condition $\phi(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$. 10

(b) Write the form of fundamental matrix of linear system with periodic coefficients. 4

Section III

5. (a) Discuss the types and nature of critical point (0, 0) of the system $\frac{dx}{dt} = x - y$, $\frac{dy}{dt} = x + 5y$. 7

(b) Construct a Liapunov function of the form $Ax^2 + By^2$, A, B being constant and determine the stability of the system $\frac{dx}{dt} = -x + 2x^2 + y^2$, $\frac{dy}{dt} = -y + xy$. 7

6. (a) Write a note on types of critical points. 7

(b) State and prove Bendixson non-existence theorem. 7

Section IV

7. (a) Solve the variance problem $J[y] = \int_1^2 \frac{\sqrt{1+y'^2}}{x} dx$, where $y(1) = 0$ and $y(2) = 1$. 7

(b) Find the extremal

$$J[y, z] = \int_0^{\pi/2} (y'^2 + z'^2 + 2yz) dx.$$

with $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 1$, $z(0) = 1$ and $z\left(\frac{\pi}{2}\right) = 1$. 7

8. (a) Find the geodesics of the sphere $\vec{r} = (a \sin\theta \cos\phi, a \sin\theta \sin\phi, a \cos\theta)$. 7

(b) Find the extremal of the functional

$$J[y] = \int_0^1 (x^2 y - y''^2) dx;\ y(0) = 0,\ y'(0) = 1,$$

$y(1) = 0$, $y'(1) = 2.5$. 7

Section V

9. (a) State the result related to reduction of order of a homogeneous system.

(b) Define Adjoint System. What is its fundamental matrix ?

(c) State Lagrange's identity.

(d) Define fundamental matrix. What is the form of fundamental matrix of system with constan coefficient ?

(e) Explain the relationship between limit cycle and periodic solutions.

(f) Define Almost Linear System.

(g) What is Brachistochrome Problem ?

$2 \times 7 = 14$