Roll No 2331106/0005 Exam Code : M-24

Subject Code—18004

M. Sc. EXAMINATION

(Batch 2020 Onwards)

(Second Semester)

MATHEMATICS

MAL-524

Ordinary Differential Equations–II

Time : 3 *Hours* *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **9** is compulsory. All questions carry equal marks.

Section I

1. (a) Show that the set of all solutions of linear homogeneous system $x' = A(t)x$ on real interval I form an *h*-dimensional vector space over complex field. **7**

(b) Write a note on reduction of order of a homogeneous system. 7

2. (a) Define Adjoint system of Linear Homogeneous (LG) system. How fundamental metrics of both the systems are related ? 7

(b) Show that the solution set $\phi_1, \phi_2, \ldots\ldots, \phi_n$ of homogeneous system $x' = A(t)x$, will be linearly dependent if Wronskian $W(\phi_1, \phi_2, \ldots\ldots, \phi_n)\ (t_0) = 0$ at some to $\in [a, b]$. 7

Section II

3. (a) Solve the system $x' = \begin{pmatrix} -2 & 1 \\ 5 & 2 \end{pmatrix} x$. Also find the solution ϕ so that $\phi(0) = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$. 7

(b) State and prove Lagrange's identity. 7

4. (a) Write a note on Linear systems with periodic coefficients. 7

(b) Explain, how a linear system can be associated with nth order linear differential equation. Discuss the relationship between solutions of both. 7

Section III

5. (a) Discuss the Liapunov's direct method to determine the asymptotic stability for non-linear systems. 7

(b) Consider non-linear autonomous system :

$$\frac{dx}{dt} = 4x - 4y - x(x^2 + y^2),$$

$$\frac{dy}{dt} = 4x + 4y - y(x^2 + y^2).$$

Use polar cordinates (r, θ) to transform the system in polar form and use Poincare-Bendixson theorem to show the existence of limits cycle between $x^2 + y^2 = \frac{1}{4}$ and $x^2 + y^2 = 16$. 7

6. (a) State the result showing type and stability of critical points of almost linear systems in terms of type of critical point of corresponding linear autonomous system. 7

(b) Show that the critical point (0, 0) of linear system $\frac{dx}{dt} = ax + by$, $\frac{dy}{dt} = cx + dy$ is node if the roots of equation $\lambda^2 - (a+d)\lambda + (ad - bc) = 0$ are real, unequal and of same sign. 7

Section IV

7. (a) Derive the Euler equation when functional depends on n unknowns and their derivatives of first order. 7

(b) Write a note on isoperimetric problem. 7

8. (a) Find the Extremal of the functional :

$$J[y] = \int_0^{\pi} y'^2 \, dx$$

subject to the conditions $y(0) = 0 = y(\pi)$,

$\int_0^{\pi} y^2 \, dx = 1$. 7

(b) Find the geodesics on the plane. 7

Section V

9. (a) Define linear system of first order differential equations. What do you mean by its solution ? 2

(b) Define fundamental set and fundamental solution. Give example. 2

(c) What is Green's formula ? 2

(d) What is method of variation of constant for non-homogeneous system ? 2

(e) Determine the nature of critical point of system $\frac{dx}{dt} = x + 3y$, $\frac{dy}{dt} = 3x + y$. 2

(f) State Poincare-Bendixson non-existence theorem. **2**

(g) Define strong and weak extremum. **2**