Roll No. ....

Exam Code : S-21

Subject Code—11203

M.Sc. EXAMINATION

(Main & Re-appear) (For Batch 2019 Onwards)

(Second Semester)

MATHEMATICS

MAL-524

Ordinary Differential Equations-II

Time : 3 Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) What is the fundamental matrix of the system $\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$?

(b) Prove that Wronskian of n linearly dependent functions on I is zero for $\forall\ t \in$ I.

(c) Explain the concept of trajectory approaching a critical point.

(d) Define perturbed linear system and simple critical points.

(e) Obtain the differential equation of the paths of $\frac{dx}{dt} = ye^x, \frac{dy}{dt} = e^x - 1$ and find its general solution.

(f) Explain the concept of calculus variations with suitable examples. 7×2=14

Unit I

2. (a) Prove that set of all solutions of a given linear system of n equations is a linear space of dimension n. 7

(b) Derive Abel's Liouville formula for a linear homogeneous system. 7

3. Explain the procedure to reduce the order of a linear homogeneous system. State the assumptions used in this process. 14

Unit II

4. (a) Find a fundamental matrix for linear system with constant coefficients. Prove your claim. 7

   (b) State and prove Representation Theorem. 7

5. (a) Derive Lagrange's Identity associated with the *n*th order linear differential operator. 7

   (b) Obtain the variance of constant formula for a non-homogeneous linear differential equation of order *n*. 7

Unit III

6. (a) Given that the roots of the characteristic equation of the linear homogeneous system are purely imaginary. Find the nature of its critical point (0, 0). 7

(b) Discuss the stability of the critical point of the differential equation of motion $m\frac{d^2x}{dt^2}+\alpha\frac{dx}{dt}+kx=0$, where $m > 0$, $\alpha \geq 0$ and $k > 0$ 7

7. (a) State and prove the basic theorem concerning the asymptotic stability of the critical point (0, 0) of the non-linear autonomous system : 7

$$\frac{dx}{dt}=P(x,y),\ \frac{dy}{dt}=Q(x,y)$$

(b) Explain limit cycles and their connection with periodic solution of a non-linear system. Provide a suitable example. 7

Unit IV

8. (a) Explain the problem of minimum surface of revolution. 7

(b) Derive Euler's equation for the functional

$$I[y_1, \ldots, y_n] = \int_a^b F(x, y_1, \ldots, y_n, y_1', \ldots, y_n')\,dx.$$

state other conditions, if any, which are required to find extremal. 7

9. (a) Define Isoperimetric problems. Among all the curves of length l ($> 2a$) in the upper half plane passing through the points $(-a, 0)$ and $(a, 0)$, find the one which together with interval $[-a, a]$ encloses the largest area. 7

(b) Explain geodesics in context of variational problems. Find the geodesics on a right circular cone of semi-vertical angle α. 7