Subject Code—15056

Dual Degree B.Sc. (Hons.) (Mathematics)–M.Sc. (Mathematics) (Fifth Semester) (Batch 2017 Onwards)

EXAMINATION

REAL ANALYSIS

BML-501

Time : 3 Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question each from Section I to Section IV. Q. No. **1** is compulsory. All questions carry equal marks.

1. (i) State fundamental theorem of integral calculus.

(ii) Prove inequality $1 \leq \int_0^1 e^{x^2} dx \leq e$

(iii) Examine the convergence of $\int_1^{\infty} \frac{dx}{x}$.

(iv) Define Euclidean Metric space.

(v) If $d(x, y) = \min\{2, |x-y|\}$, show that d is a metric for R.

(vi) Show that isometry is a uniformly continuous function.

(vii) Define Bolzano-Weierstrass property. **14**

Section I

2. (a) Show that the function f defined by $f(x) = x, x \in [0, 1]$ is integrable and $\int_0^1 f(x)\,dx = \frac{1}{2}$. 7

(b) Show that the function f defined by
$$f(x) = \frac{1}{m^{n-1}}, \text{ if } \frac{1}{m^n} < x \le \frac{1}{m^{n-1}}, x \in N$$
when m is an integer ≥ 2, is integrable on $[0, 1]$. Also evaluate $\int_0^1 f.dx$. 7

3. (a) If f and g are two integrable functions on $[a, b]$ then fg is integrable on $[a, b]$. 7

(b) Verify first mean value theorem for the integral :

$$\int_0^2 \left(4x^3 - 3x^2 - 4x\right) dx.$$

7

Section II

4. (a) Examine the convergence of the improper integral $\int_{-1}^{1} \frac{dx}{(2-x)\sqrt{1-x^2}}$. 7

(b) Show that $\int_0^{\pi/2} \sin^{p-1} x \cos^{q-1} dx$ exists if and only if $p > 0$ and $q > 0$. 7

5. (a) Test the convergence of :

$$\int_0^1 \frac{\sin \frac{1}{x}}{\sqrt{x}} . dx.$$

7

(b) Show that :

$$\int_0^\infty \frac{\sin ax \sin bx}{x}\, dx$$

converges to $\frac{1}{2}\log\frac{a+b}{a-b}$, where $a>b>0$. 7

Section III

6. (a) Show that any metric space (X, d) bounded or not, can be converted into a bounded metric space (X, d^*) where

$$d^*(x, y) = \frac{d(x, y)}{1+d(x, y)}.$$ 7

(b) If A and B are subsets of a metric space (X, d), then show that : 7

(i) $(A \cap B)^\circ = A^\circ \cap B^\circ$

(ii) $A^\circ \cup B^\circ \subset (A \cup B)^\circ$.

7. (a) State and prove Cantor's intersection theorem. 7

(b) Show that every second conuntable metric space is separable. 7

Section IV

8. (a) Let (X, d) and (Y, d^*) be metric spaces and f is function of X into Y. Then show that f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y. 7

(b) Show that closed subspace of a compact metric space is compact. 7

9. (a) Prove that every sequentially compact space is countably compact. 7

(b) Show that two closed subsets of a metric space are separated iff they are disjoint. 7