Roll No. ....

Exam Code : M-24

Subject Code—18059

Dual Degree B.Sc. (Hons.)

EXAMINATION

(Batch 2017 Onwards)

(Sixth Semester)

MATHEMATICS

BML-601

Real and Complex Analysis

Time : 3 Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (a) If $u = x(1 - y)$, $v = xy$, then show that :

$$\frac{\partial(u,v)}{\partial(x,y)} = \left(\frac{\partial(x,y)}{\partial(u,v)}\right)^{-1}. \qquad 2$$

(b) Prove that $\Gamma(n) = (n-1)\,\Gamma(n-1)$ for real number $n > 1$. 2

(c) Change the order of integration of

$$\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx.$$ 2

(d) Define Fourier series for a bounded function. 2

(e) Show that $f(z) = z^2$ is uniformly continuous in the region $|z| < 1$. 2

(f) Find k such that $f(z) = r^2 \cos 2\theta + i\, r^2 \sin k\theta$ is analytic. 2

(g) Define conformal mapping. 2

Section I

2. (a) Show that the functions $u = x^2 + y^2 + z^2$, $v = x + y + z$, $w = xy + yz + zx$ are functionally dependent. Find the relation between them. 7

(b) Prove that :

$$B(m,n)=\frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}, \quad \text{where } m > 0,$$

$n > 0$. 7

3. (a) Show that : $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dx\,dy=\frac{\pi}{4}$. 7

(b) Using Dirichlet's theorem, find the volume bounded by the surface

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^4}{c^4}=1.$$ 7

Section II

4. (a) Find the Fourier series of x^2 in $(-\pi, \pi)$.

Also prove that $\frac{\pi^4}{90}=1+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}$.

7

(b) Find the Fourier series for the function $f(x) = |x|, -\pi \leq x \leq \pi$. Also prove that

$$\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots..$$ 7

5. Find the half range sine series and cosine series for the function : **14**

$$f(x) = \begin{cases} x & \text{for} \quad 0 < x < \dfrac{\pi}{2} \\ \pi - x & \text{for} \quad \dfrac{\pi}{2} < x < \pi \end{cases}$$

Section III

6. (a) If $f(z)$ is a regular function, then prove that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4\,|f'(2)|^2$. 7

(b) If $f(z) = u + iv$ is an analytic function of z, find $f(z)$ if $u - v = (x - y)(x^2 + 4xy + y^2)$. 7

7. (a) Show that real and imaginary parts of an analytic function are harmonic functions. 7

(b) In a two dimensional fluid flow, the stream function is given by $\psi = -\dfrac{y}{x^2 + y^2}$. Find the velocity potential φ. 7

Section IV

8. (a) Find the image of infinite strip $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. Show the region graphically. 7

(b) Show that cross-ratio remains invariant under mobius transformation. 7

9. (a) Find the image of $x^2 = y^2 - 4y + 2 = 0$ under the mapping $w = \frac{z-i}{iz-1}$. 7

(b) Find all the Mobius transformations which map the half plane $I(z) = \geq 0$ into circle $|w| \leq 1$. 7