Subject Code—15059

Dual Degree B.Sc. (Hons.) Mathematics-M.Sc. Mathematics (Fifth Semester) (Batch 2017 Onwards)

EXAMINATION

SEQUENCE AND SERIES (Elective-I)

BML-505

Time : 3 *Hours* *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

1. (a) Prove that the intersection of an arbitrary family of open sets may not be an open set. **2**

(b) Discuss the boundedness of the sequence $< a_n >$, where $a_n = \left(1+\frac{1}{n}\right)^n$. **2**

(c) Prove that a real number l is a limit point of a sequence $< a_n >$ if and only if there exists a subsequence of $< a_n >$ converging to l. 2

(d) State Cauchy's root test. 2

(e) Examine the convergence of the following series : 2

$$\frac{1}{2}+\frac{1.3}{2.4}+\frac{1.3.5}{2.4.6}+....$$

(f) Give an example of an infinite series which is convergent but not absolutely convergent. 2

(g) State Pringsheim's theorem. 2

Unit I

2. (a) Show that the set of reals is not a compact set. 7

(b) Prove that the union of finite number of closed sets is a closed set. 7

3. (a) Show that every set satisfying the Heine Borel property is a compact set. 7

(b) If A and B are subsets of R, then prove that $(A \cup B)' = A' \cup B'$. 7

Unit II

4. (a) Let $< a_n >$ be a sequence such that $a_n \neq 0$ for all $n \in N$, where N is the set of all natural numbers, and $\frac{a_{n+1}}{a_n} \to l$. Prove that if $|l| < 1$, then $\lim_{n\to\infty} a_n = 0$. 7

(b) Show that the series $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots.$ converges to 1. 7

5. (a) Find : 7

$$\lim_{n\to\infty} \left[\frac{(n+1)(n+2)\ldots.(n+n)}{n^n}\right]^{\frac{1}{n}}.$$

(b) Test the convergence of the following series : 7

$$\sum_{n=1}^{\infty} \sqrt{n^4+1}-\sqrt{n^4-1}$$

Unit III

6. (a) State and prove D-Alembert's ratio test. 7

(b) Test the convergence of the following series : 7

$$1+\frac{2^2}{3^2}+\frac{2^2.4^2}{3^2.5^2}+\frac{2^2.4^2.6^2}{3^2.5^2.7^2}+.....$$

7. (a) Test the convergence of the following series : 7

$$x^2+\frac{2^2}{3.4}x^4+\frac{2^2.4^2}{3.4.5.6}x^6+\frac{2^2.4^2.6^2}{3.4.5.6.7.8}x^8+.... \ (x>0)$$

(b) Discuss the convergence of the following series : 7

$$\frac{\alpha}{\beta}+\frac{1+\alpha}{1+\beta}+\frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)}+....$$

Unit IV

8. (a) Show that the series $x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots.$ is absolutely convergent if and only if $|x|<1$ but conditionally convergent for $[x]=1$. 7

(b) State and prove Abel's test. 7

9. (a) Discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^n \sin n\alpha}{n^p}$, $p>0$ for all real α. 7

(b) Show that the series $\left(1-\frac{1}{2}\right)+\left(1-\frac{3}{4}\right)+\left(1-\frac{7}{8}\right)+\ldots.$ is convergent, but when the parentheses are removed, it oscillates. 7