Subject Code—12037

B. Sc. (Hons.) EXAMINATION

(Batch 2016 Onwards)

(Fourth Semester)

MATHEMATICS

BML-406

Special Function–II

Time : 2 *Hours* *Maximum Marks* : 36

Note : Attempt *Three* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Write Laguerre's equation and Laguerre's polynomial of order n. **2**

(b) Prove that :

$$L_3(x) = \frac{1}{3!}\left(6 - 18x + 9x^2 - x^3\right). \quad 2$$

(c) Write the generating function for the associated Laguerre's polynomial. 2

(d) Define Kummer's theorem. 2

(e) Prove that :

$$\log_e(1+x) = x\,{}_2F_1(1, 1; 2; -x). \quad 2$$

(f) Write orthogonal property of Laguerre's Polynomial. 2

Section I

2. (a) Prove that :

$$L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}\left(x^n e^{-x}\right). \quad 6$$

(b) Prove that :

$$\int_0^\infty L_n(t)\,dt = \frac{1}{s}\left(1 - \frac{1}{s}\right)^n. \quad 6$$

3. (a) Prove that :

$$\int_0^\infty e^{-x} L_m(x) L_n(x)\,dx = \delta_{mn}. \quad 6$$

(b) Prove that :

$$L_{n-1}^{\alpha}(x) + L_{n}^{\alpha-1}(x) = L_{n}^{\alpha}(x). \qquad 6$$

Section II

4. (a) Prove that :

$$F(\alpha, \beta; x) = \frac{\Gamma(\beta)}{\Gamma(\alpha)\Gamma(\beta-\alpha)} \int_0^1 (1-t)^{\beta-\alpha-1} t^{\alpha-1} e^{xt} dt,$$

for $\beta > \alpha > 0$. 6

(b) Prove that :

$$F(\alpha, \beta; \beta; x) = (1-x)^{-\alpha}. \qquad 6$$

5. (a) State and prove integral representation for the hypergeometric function. 6

(b) Show that :

$$\frac{d}{dx}\left(F(\alpha, \beta; \gamma; x)\right) + \frac{\alpha\beta}{\gamma} F(\alpha+1, \beta+1; \gamma+1; x).$$