Roll No. ..

Exam Code : D-22

Subject Code—11734

Dual Degree B. Sc. (Hons.) (Mathematics)-M.Sc. (Mathematics)

EXAMINATION

(Batch 2016 Onwards)

(Third Semester)

SPECIAL FUNCTIONS-I

BML-306

Time : 2 *Hours* | *Maximum Marks* : 36

Note : Attempt *Three* questions in all including Q. No. **1**, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Show that $x = 0$ is an ordinary point of

$$(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$$ but $x = 1$ is a regular singular point.

(b) Define Beta and Gamma functions.

(c) Write orthogonality relation of Bessel's function.

(d) Verify that Legendre polynomial $P_3(x) = \frac{1}{2}(5x^3 - 3x)$ satisfies the Legendre equation when the parameter n is equal to 3.

(e) Find the value of :

$$\int_{-\infty}^{\infty} e^{-x^2} H_2(x) H_3(x) dx.$$

(f) Prove that :

$$\int_{-1}^{1} P_n(x)(1 - 2tx + t^2)^{-1/2} dx = \frac{2t^n}{2n+1}.$$

2×6=12

Unit I

2. (a) Find the power series solution of the differential equation :

$$(x^2 - 1)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 0 \text{ about } x = 0.$$

(b) Solve the equation :

$$\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}+\left(1-\frac{1}{9x^2}\right)y=0 \text{ in terms of}$$

Bessel's function.

6,6

3. (a) Show that :

(i) $$\frac{d}{dx}(x^n J_n(x)) = x^n J_{n-1}(x)$$

(ii) $$\frac{d}{dx}(x^{-n} J_n(x)) = -x^{-n} J_{n+1}(x).$$

(b) Show that :

6,6

$$J_4(x)=\left(\frac{84}{x^3}-\frac{8}{x}\right)J_1(x)+\left(1-\frac{24}{x^2}\right)J_0(x).$$

Unit II

4. (a) Derive Legendre's polynomials from Rodriue formula.

(b) Prove that :

$$\int_{-1}^{1} x^2 P_{n+1}(x) P_{n-1}(x) dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}.$$

Hence deduce the value of

$$\int_{0}^{1} x^2 P_{n+1}(x) P_{n-1}(x) dx.$$ **6,6**

5. (a) State and prove generating function of Hermite's polynomial.

(b) Explain orthogonal property of Hermite's polynomial. **6,6**