

Roll No. ....

Exam Code : M-23

Subject Code—12036

**B. Sc. (Hons.) EXAMINATION**

(Batch 2016 Onwards)

(Fourth Semester)

MATHEMATICS

BML-405

Operations Research—I

*Time : 3 Hours*

*Maximum Marks : 70*

**Note :** Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

**(Compulsory Question)**

1. (a) What is Operations Research ? Explain  
in brief.

2

P.T.O.

(5-40-19-0523) J-12036

(b) Write all the conditions for standard form of Linear Programming Problem (LPP) with example. 2

(c) Does the following LPP has a feasible solution ?

$$\text{Max. : } Z = x_1 + x_2$$

Subject to :

$$2x_1 - 3x_2 \geq 4,$$

$$x_2 - 2x_1 \geq 0,$$

$$x_1, x_2 \geq 0. \quad 2$$

(d) Obtain the dual of the following LPP :

$$\text{Min. : } Z = x_1 + x_2 + x_3$$

Subject to the constraints :

$$x_1 - 3x_2 + 4x_3 = 5,$$

$$x_1 - 2x_2 \leq 3,$$

$$2x_2 - x_3 \geq 4,$$

$$x_1, x_2 \geq 0$$

and  $x_3$  is unrestricted. 2

(e) With reference to a transportation problem define the following terms : 2

(i) Feasible solution

- (ii) Basic feasible solution
- (iii) Optimal solution.
- (f) Write a short note on Assignment problem. 2
- (g) Explain zero-sum two-person game giving suitable example. 2

### Unit I

- 2. (a) Write the scope and any *ten* applications of operations research. 7
- (b) Explain the meaning of a Linear Programming Problem, stating its advantages and write its limitations. 7
- 3. (a) A leather company manufactures two types of belts  $B_1$  and  $B_2$ . Each of these sell at a profit of Rs. 2 per belt. Each belt is processed on two machines  $M_1$  and  $M_2$ . The  $B_1$  type belt requires one minute of processing time on machine

$M_1$  and two minutes on machine  $M_2$ . The  $B_2$  type belt requires one minute of processing time on each of the machines  $M_1$  and  $M_2$ . While machine  $M_1$  is available for a maximum of 6 hours and 40 minutes, machine  $M_2$  is available for 10 hours on all working days. Formulate the given problem as an LPP and solve it using the graphical method. 7

- (b) Solve the Linear Programming Problem (LPP) by graphical method :

$$\text{Min. : } Z = 3x_1 + 5x_2$$

Subject to :

$$-3x_1 + 4x_2 \leq 12,$$

$$2x_1 - x_2 \geq -2,$$

$$2x_1 + 3x_2 \geq 12,$$

$$x_1 \leq 4,$$

$$x_2 \geq 2,$$

$$x_1, x_2 \geq 0.$$

7

## Unit II

4. (a) Describe a computational procedure of the simplex method for the solution of a maximization LPP. 7

- (b) Solve the following LPP using the Simplex method :

$$\text{Max. : } Z = 4x_1 + 6x_2$$

Subject to :

$$2x_1 + 3x_2 \leq 6,$$

$$x_1 - x_2 \leq 1,$$

$$x_1, x_2 \geq 0.$$

Further, determine the alternate optimal solution if it exists. 7

5. (a) Solve the following LPP using the Big-M method :

$$\text{Max. : } Z = 4x_1 + 5x_2 - 3x_3$$

Subject to :

$$x_1 + x_2 + x_3 = 10,$$

$$x_1 - x_2 \geq 1,$$

$$2x_1 + 3x_2 + x_3 \leq 40,$$

$$x_1, x_2, x_3 \geq 0. \quad 7$$

- (b) Using two-phase method solve the following LPP :

$$\text{Max. : } Z = 5x_1 - 2x_2 + 3x_3$$

Subject to :

$$2x_1 + 2x_2 - x_3 \geq 2,$$

$$3x_1 - 4x_2 \leq 3,$$

$$x_2 + 3x_3 \leq 5,$$

$$x_1, x_2, x_3 \geq 0.$$

7

### Unit III

6. (a) Describe primal-dual pair relationship. Prove that dual of the dual of a given primal is the primal.

7

- (b) Apply the principle of duality to solve the Linear Programming Problem (LPP) :

$$\text{Max. : } Z = 3x_1 - 2x_2$$

Subject to :

$$x_1 + x_2 \leq 5,$$

$$x_1 \leq 4,$$

$$1 \leq x_2 \leq 6,$$

$$x_1, x_2 \geq 0.$$

7

7. (a) Determine the optimal basic feasible solution to the following transportation problem using the MODI method : 7

|          |                | To             |                |                |                | Available |
|----------|----------------|----------------|----------------|----------------|----------------|-----------|
|          |                | D <sub>1</sub> | D <sub>2</sub> | D <sub>3</sub> | D <sub>4</sub> |           |
| From     | O <sub>1</sub> | 5              | 3              | 6              | 2              | 19        |
|          | O <sub>2</sub> | 4              | 7              | 9              | 1              | 37        |
|          | O <sub>3</sub> | 3              | 4              | 7              | 5              | 34        |
| Required |                | 16             | 18             | 31             | 25             |           |

- (b) Find an optimum basic feasible solution to the following transportation problem (Use dummy destinations, if needed) : 7

|          |                | Dertination    |                |                |                |                | Available |
|----------|----------------|----------------|----------------|----------------|----------------|----------------|-----------|
|          |                | D <sub>1</sub> | D <sub>2</sub> | D <sub>3</sub> | D <sub>4</sub> | D <sub>5</sub> |           |
| Orgin    | O <sub>1</sub> | 5              | 4              | 8              | 6              | 5              | 600       |
|          | O <sub>2</sub> | 4              | 5              | 4              | 3              | 2              | 400       |
|          | O <sub>3</sub> | 3              | 6              | 5              | 8              | 4              | 1000      |
| Required |                | 450            | 400            | 200            | 250            | 300            |           |

## Unit IV

8. (a) Find men are available to do five different jobs. From past records, the time (in hours) that each man takes to do each job is known and given in the following table :

|     |       | Jobs  |       |       |       |       |
|-----|-------|-------|-------|-------|-------|-------|
|     |       | $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ |
| Man | $M_1$ | 2     | 9     | 2     | 7     | 1     |
|     | $M_2$ | 6     | 8     | 7     | 6     | 1     |
|     | $M_3$ | 4     | 6     | 5     | 3     | 1     |
|     | $M_4$ | 4     | 2     | 7     | 3     | 1     |
|     | $M_5$ | 5     | 3     | 9     | 5     | 1     |

Find the assignment of men to jobs that will minimize the total time taken. 7

- (b) A company has 5 jobs to be done. The following matrix shows the return in rupees on assigning  $i$ th ( $i = 1, 2, 3, 4, 5$ ) machine to the  $j$ th job ( $j = A, B, C,$

D, E). Assign the five jobs to the five machines so as to maximize the total expected profit.

7

|         |   | Job |    |    |    |   |
|---------|---|-----|----|----|----|---|
|         |   | A   | B  | C  | D  | E |
| Machine | 1 | 5   | 11 | 10 | 12 | 4 |
|         | 2 | 2   | 4  | 6  | 3  | 5 |
|         | 3 | 3   | 12 | 5  | 14 | 6 |
|         | 4 | 6   | 14 | 4  | 11 | 7 |
|         | 5 | 7   | 9  | 8  | 12 | 5 |

9. (a) Solve the following game using the graphical method :

7

|          |                | Player B       |                |
|----------|----------------|----------------|----------------|
|          |                | B <sub>1</sub> | B <sub>2</sub> |
| Player A | A <sub>1</sub> | 1              | 2              |
|          | A <sub>2</sub> | 5              | 6              |
|          | A <sub>3</sub> | -7             | -9             |
|          | A <sub>4</sub> | -4             | -3             |
|          | A <sub>5</sub> | 2              | 1              |

- (b) Solve the following game using the linear programming method :

7

|          |       | Player B |       |       |
|----------|-------|----------|-------|-------|
|          |       | $B_1$    | $B_2$ | $B_3$ |
| Player A | $A_1$ | 3        | -4    | 2     |
|          | $A_2$ | 1        | -3    | -7    |
|          | $A_3$ | -2       | 4     | 7     |